Economics and Computation

Ad Auctions and Other Stories

Christopher A. Wilkens

UC Berkeley

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Why mix Economics and Theoretical Computer Science?

Alan Turing, 1936:

Introduced the Turing Machine as a tool to understand the limits of *Logic*.

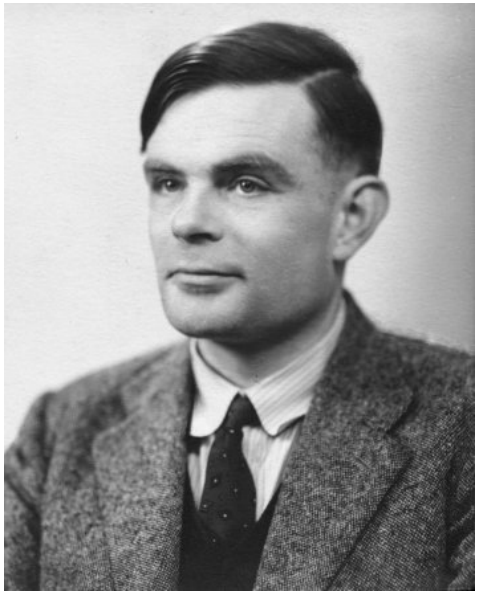

Image Source: Wikipedia

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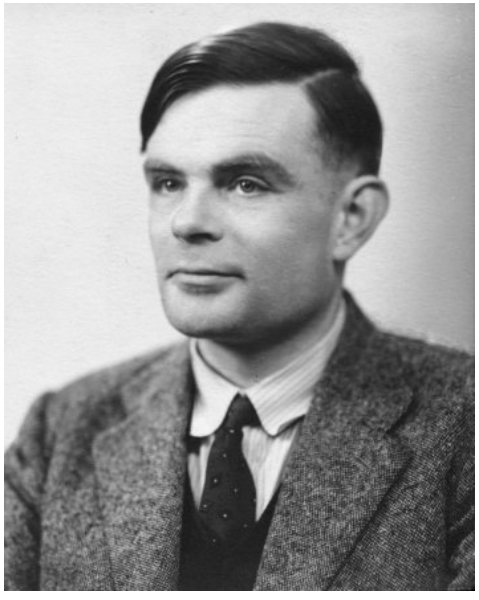
Introduced the Turing Machine as a tool to understand the limits of *Logic*.

Looking back...

The limits of Logic cannot be fully understood *without* computational ideas!



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Economics today:

Many important questions about complex economic systems require a computational perspective.

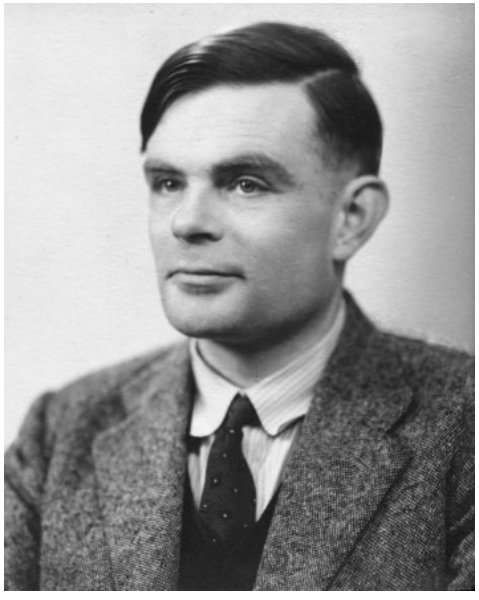


Image Source: Wikipedia

Today's Talk

Sponsored Search Auctions

- *First-Price Auctions:*
  How can we design first-price auctions that perform well?
- *Coopetitive Ad Auctions:*
  Recognizing complexity may be important for performance.

Market Equilibria

- *Complexity Equilibria in Markets*
  Computational complexity begets stability.

The Sponsored Search Auction

Sponsored Search — History

A long time ago in a galaxy far, far away...

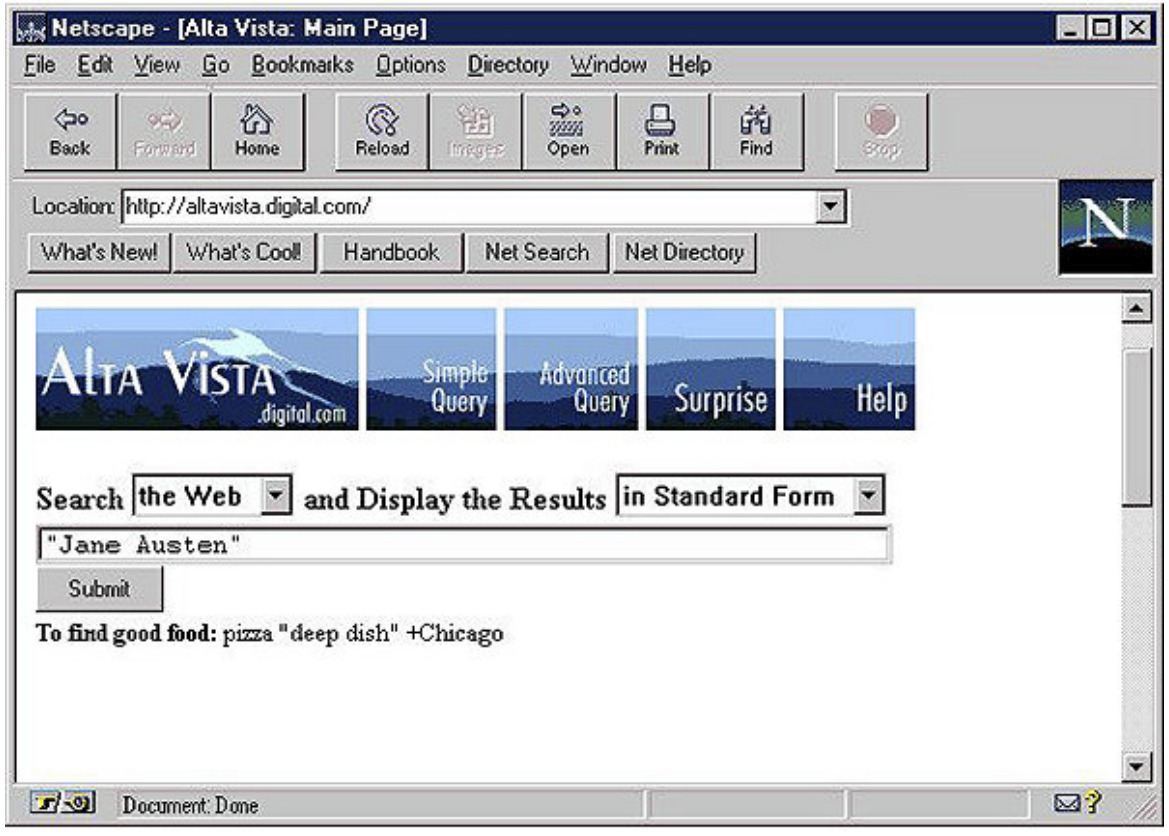


Image Source: Computer History Museum

Sponsored Search — History

Idea: Willingness to pay is a proxy for relevance and quality.

Sponsored Search — History

1996: GoTo.com introduces paid search.

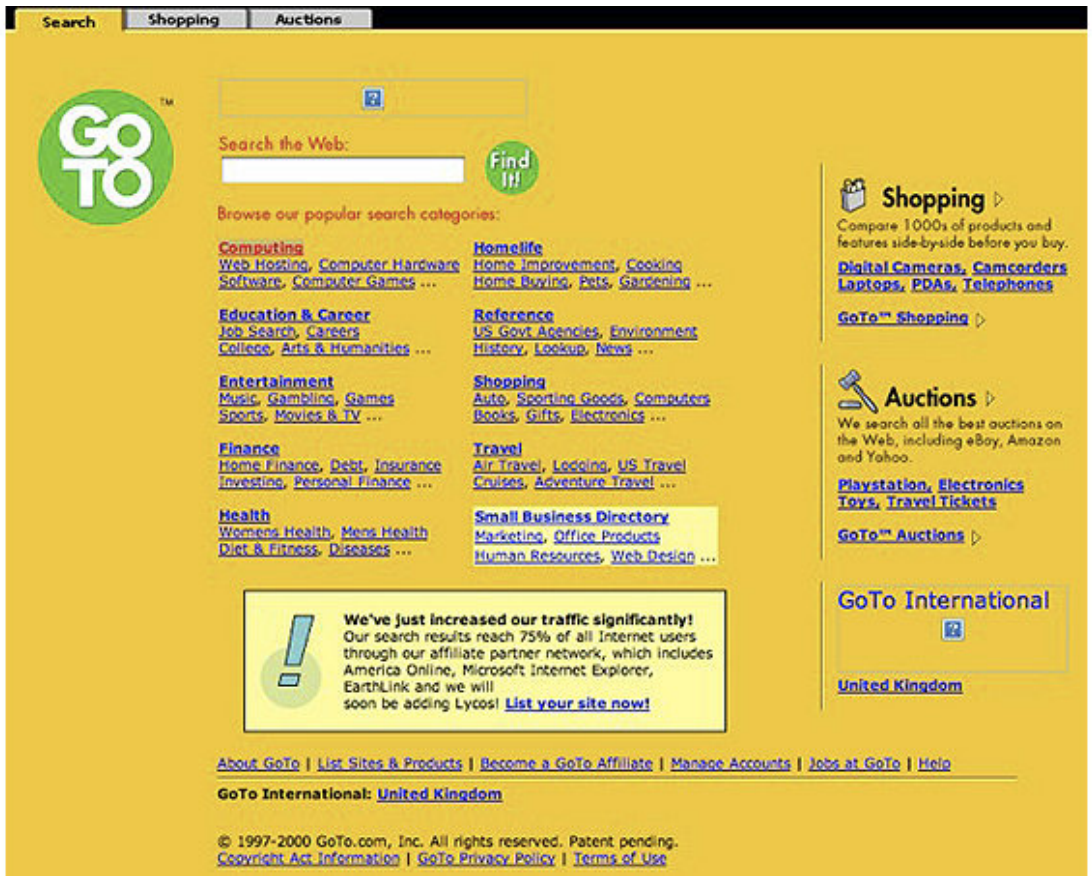


Image Source: Computer History Museum

Sponsored Search — History

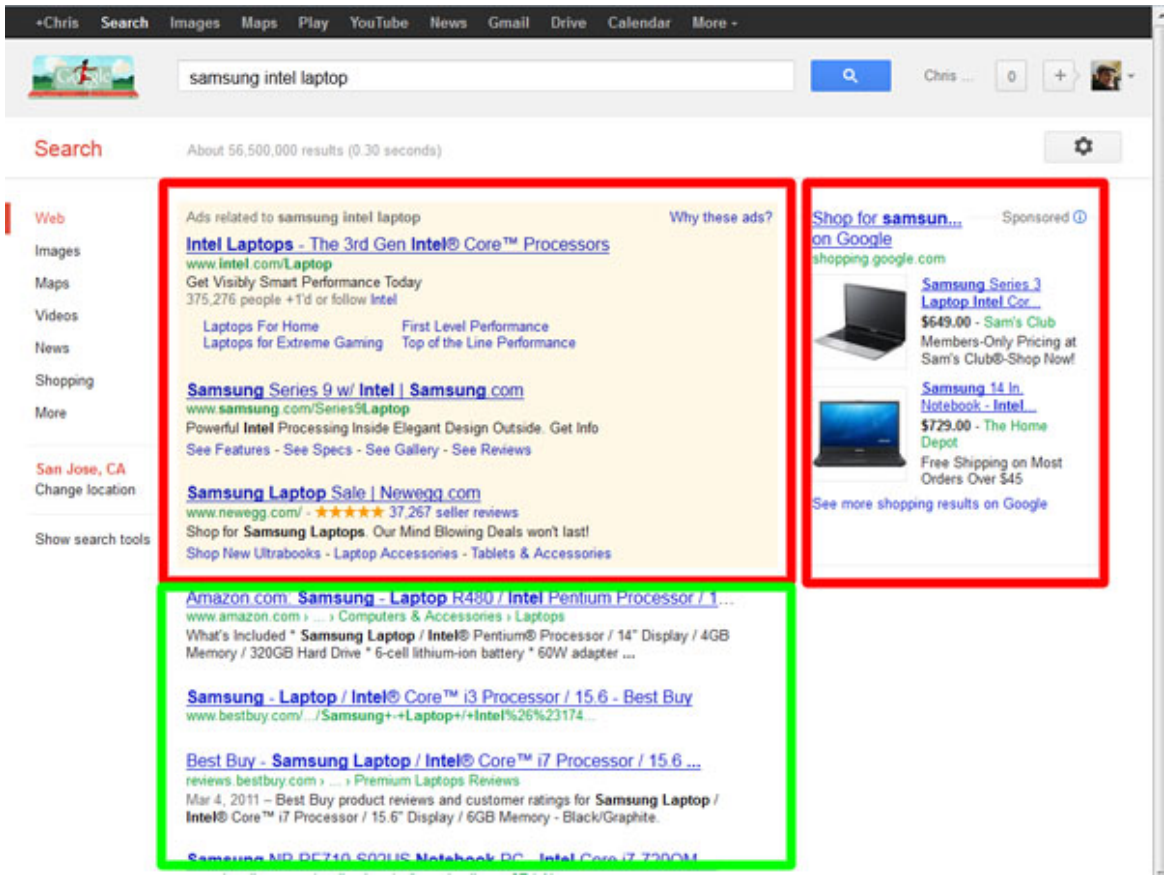
The GoTo.com Model:

Intel bids \$2 for "Intel Laptop". When user searches for "Intel Laptop"...

- Results for "Intel Laptop" sorted by bid.
- Intel pays \$2 if user clicks on link to `www.intel.com` (pay-per-click, PPC)

Sponsored Search — History

Today: Ads shown alongside organic results.



Sponsored Search — History

1996 GoTo.com launches first successful search engine with sponsored results.

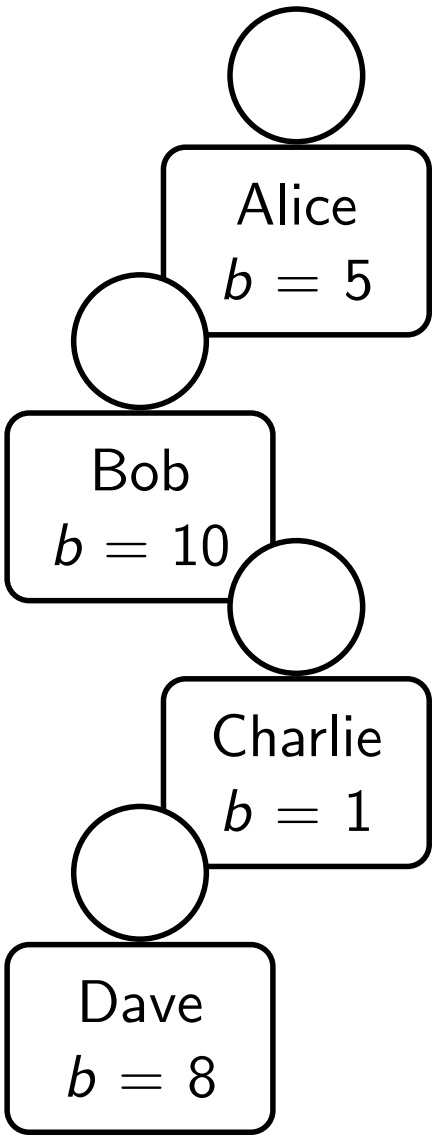
GoTo.com partners with Yahoo!, MSN, AOL, and others to show sponsored results alongside organic ones.

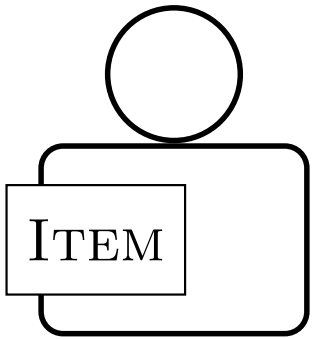
2000 Google launches Adwords platform

2001 GoTo.com changes name to Overture.com.

2003 Overture purchased by Yahoo!

Aside — Single-Item Auctions





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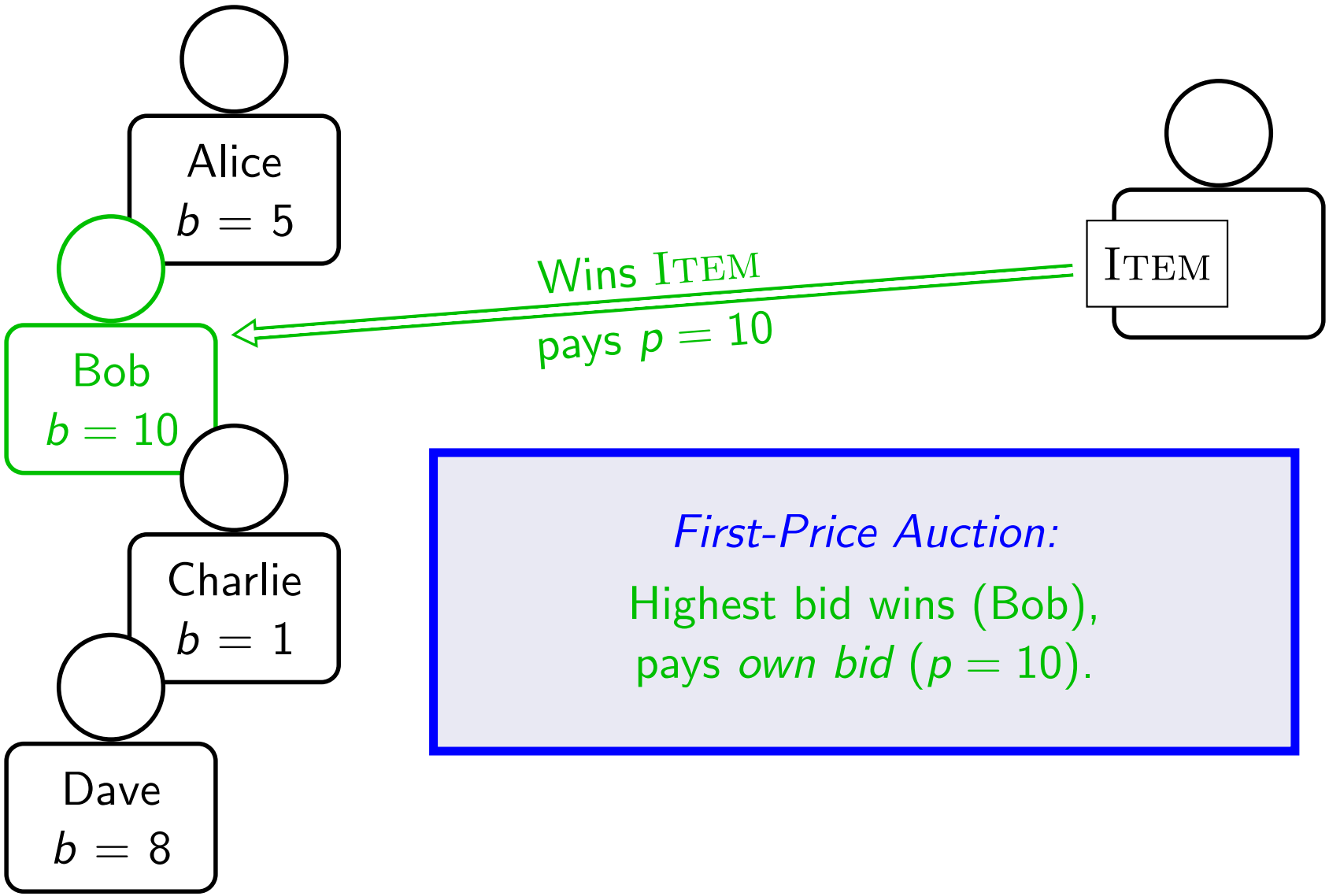
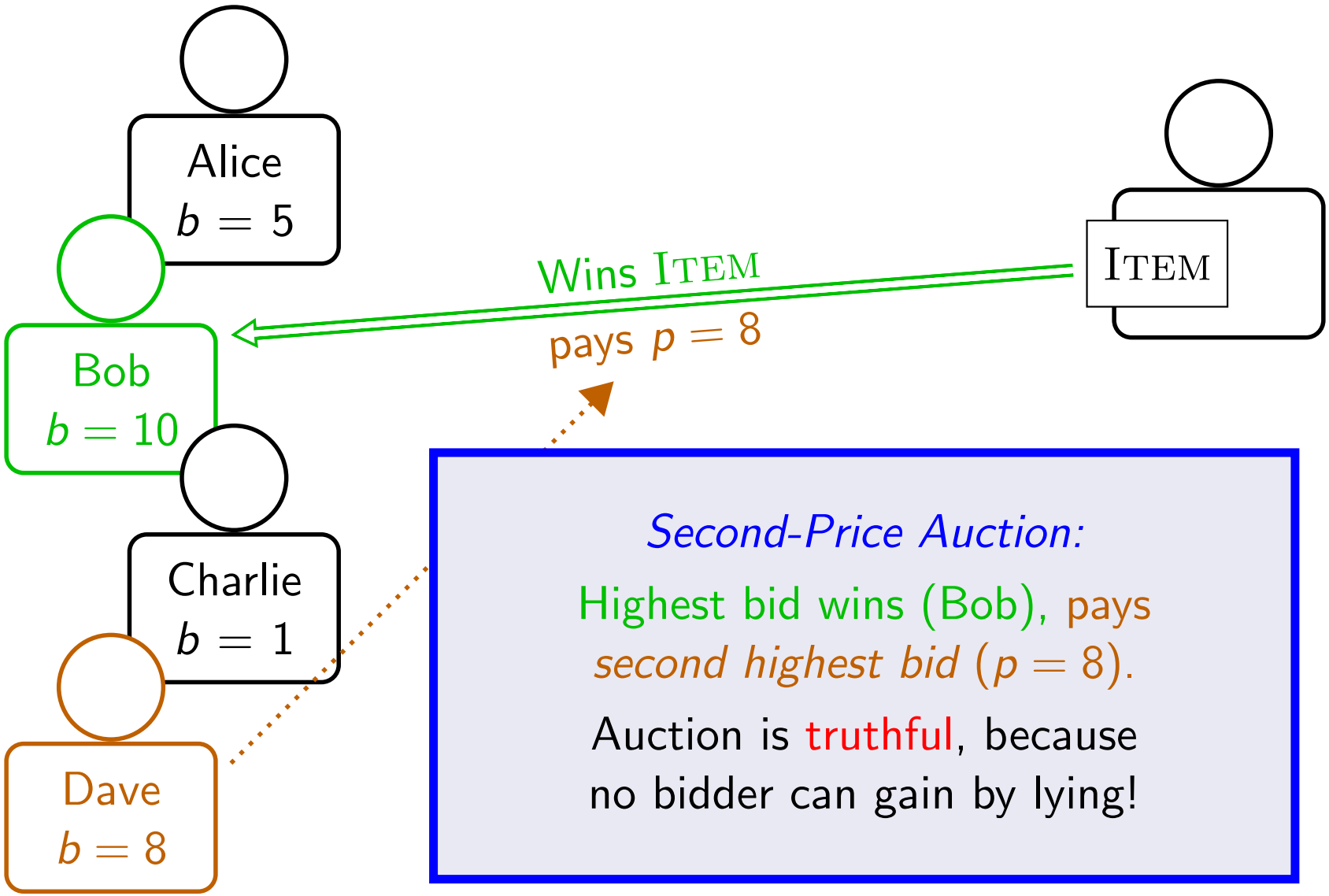


First-Price Auction:

Highest bid wins (Bob),
pays *own bid* ($p = 10$).

Aside — Single-Item Auctions



Second-Price Auction:

Highest bid wins (Bob), pays *second highest bid* ($p = 8$).

Auction is truthful, because no bidder can gain by lying!

Aside — The VCG Auction

Question: What about truthfulness in complex auctions?

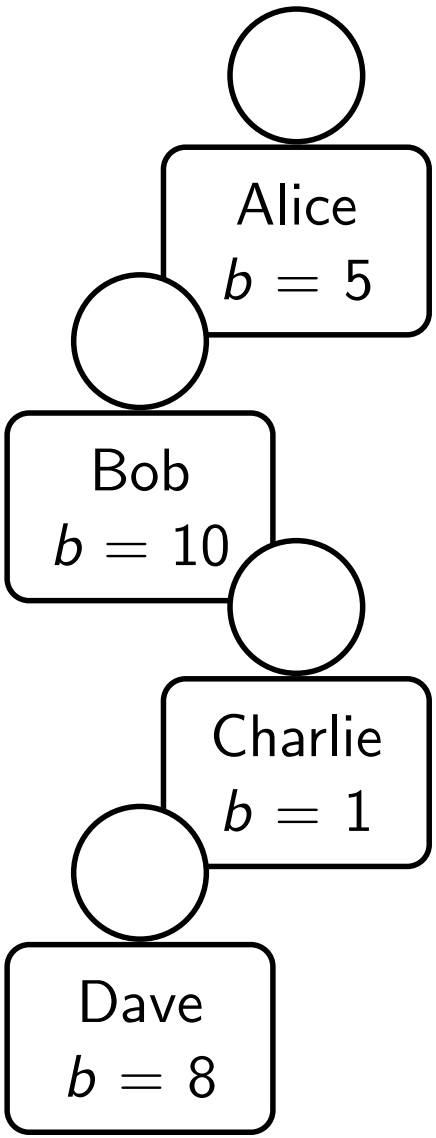
The Vickrey-Clarke-Groves (VCG) Auction:

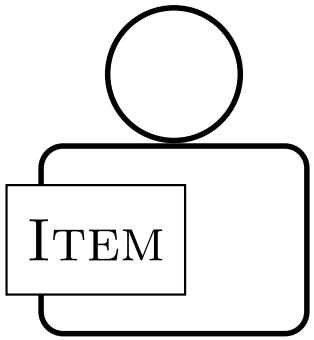
- Pick the socially optimal allocation of goods to bidders.
- Payment is negative externality:

$$p_i = \textsc{Welfare}_{-i}[\text{excluding } i] - \textsc{Welfare}_{-i}[\text{including } i]$$

$$\left(\textsc{Welfare}_{-i} = \sum_{j \neq i} [j\text{'s value for chosen allocation}] \right)$$

Aside — The VCG Auction





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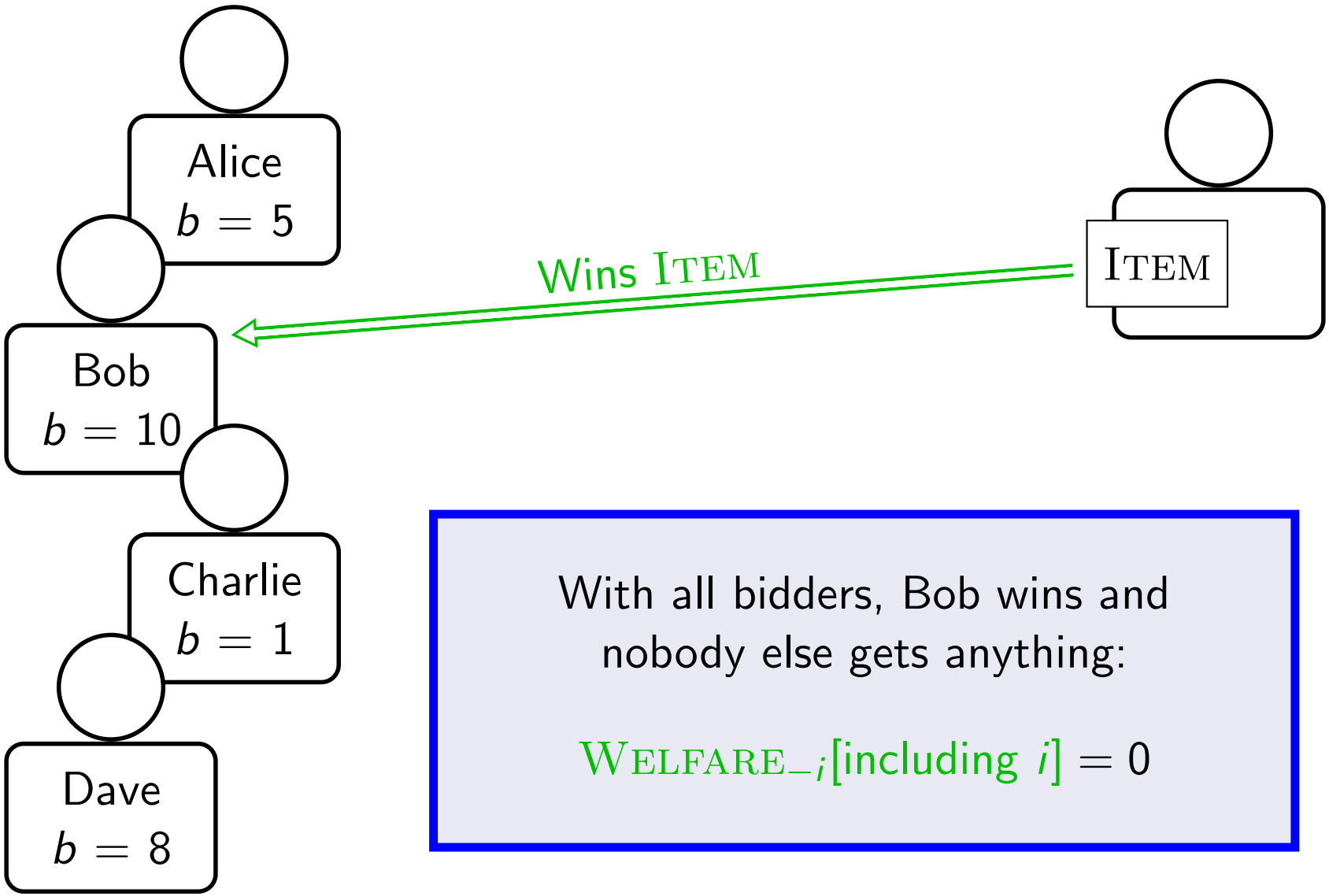
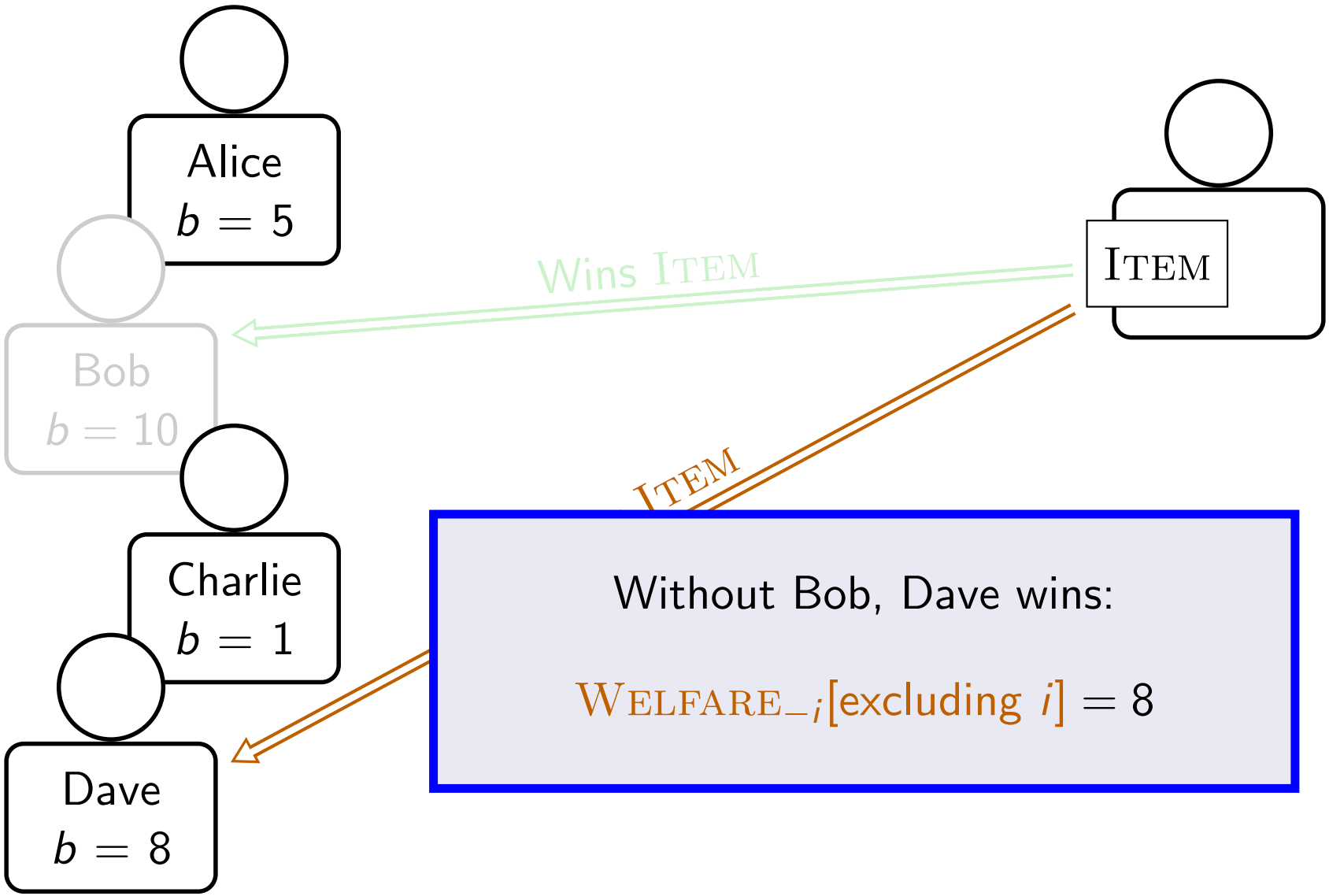


With all bidders, Bob wins and nobody else gets anything:

$\textsc{Welfare}_{-i}[\text{including } i] = 0$

Aside — The VCG Auction



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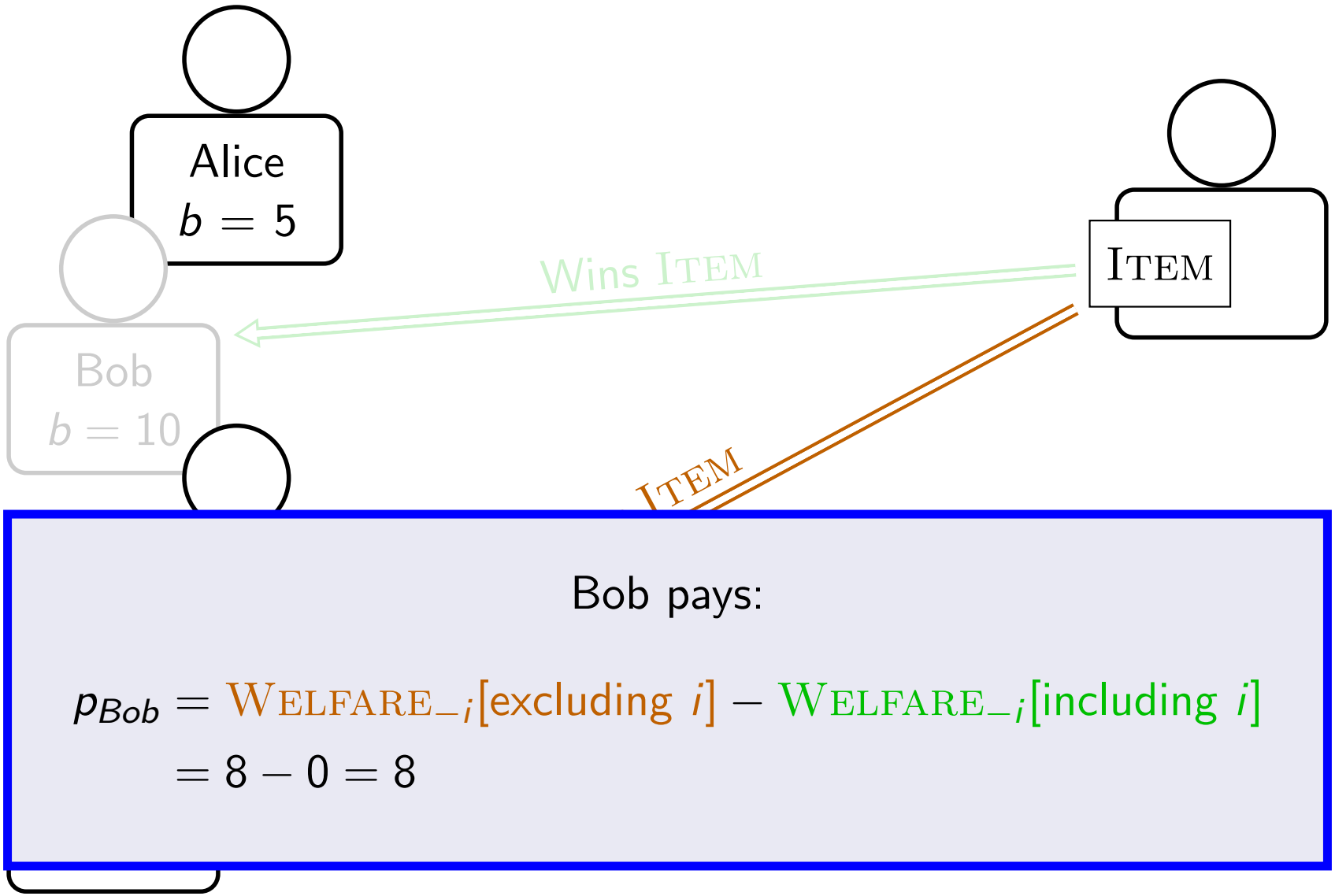


Bob pays:

$$p_{Bob} = \textsc{Welfare}_{-i}[\text{excluding } i] - \textsc{Welfare}_{-i}[\text{including } i]$$
$$= 8 - 0 = 8$$

Sponsored Search — History of the Auction

1996 — GoTo.com uses generalized first-price auction (GFP) to sell sponsored results. Bids are unstable.

2000 — Google's Adwords program sells advertising through monthly contracts.

2002 — Google introduces generalized second-price (GSP) auction for Adwords. Features:

- Results "ranked by revenue."
- Payment is "next highest bid."

Sponsored Search — History of the Auction

Generalized Second-Price (GSP) Auction:

Company i bids $\$b_i$ for QUERY...

- Click-through-rate (CTR) is the likelihood a user clicks on i's ad when shown in slot j:

$$c_{i,j} = \alpha_j \times \beta_i$$

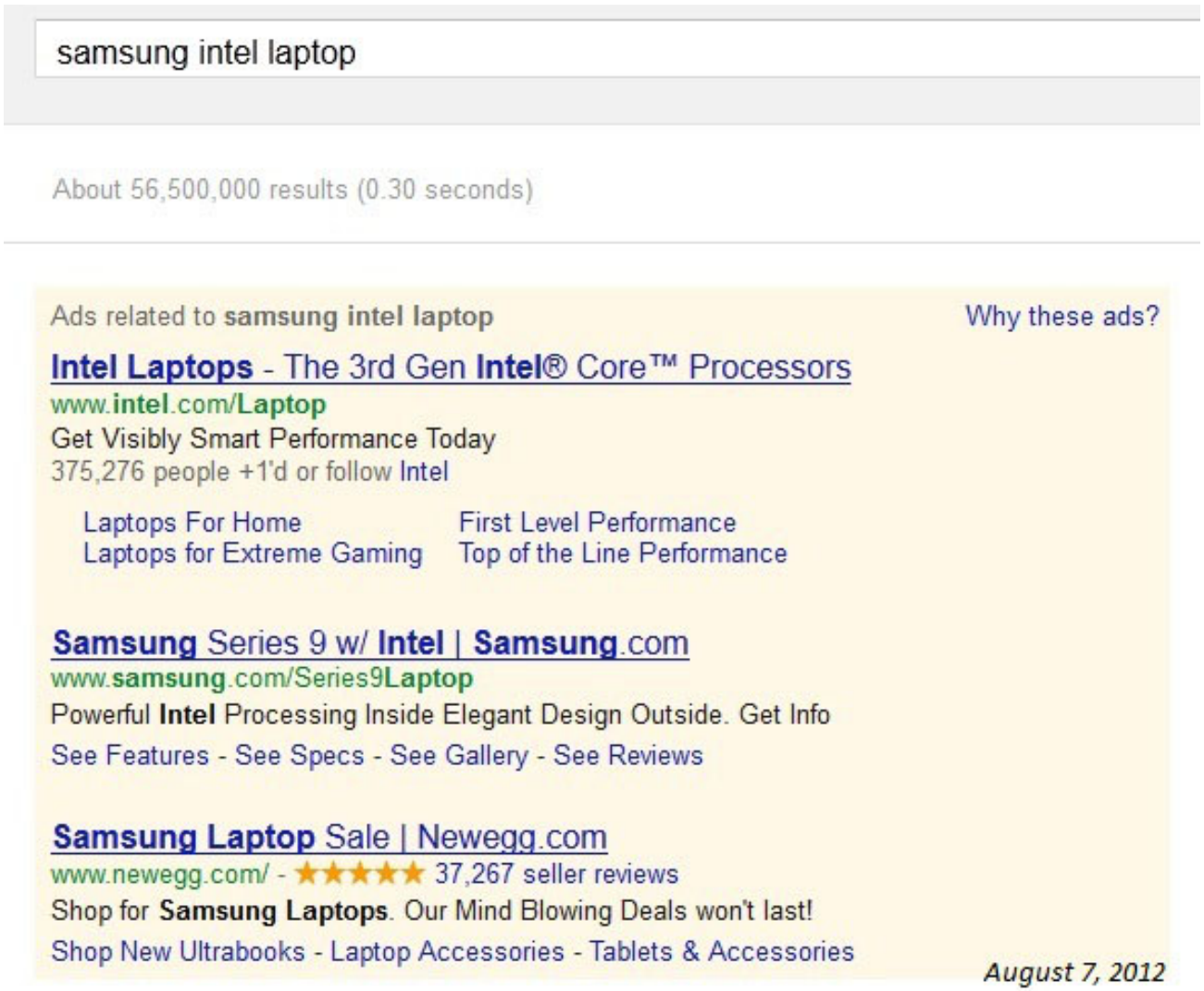
- Expected revenue is

$$R = \sum_i c_{i,j(i)} p_i = \sum_i \alpha_{j(i)} \beta_i p_i$$

- Sort results by $\beta_i \times b_i$.
- Per-click payment p_i is minimum bid required for current rank:

$$p_i = \frac{\beta_{i+1}}{\beta_i} b_{i+1}$$

Sponsored Search — GSP Example



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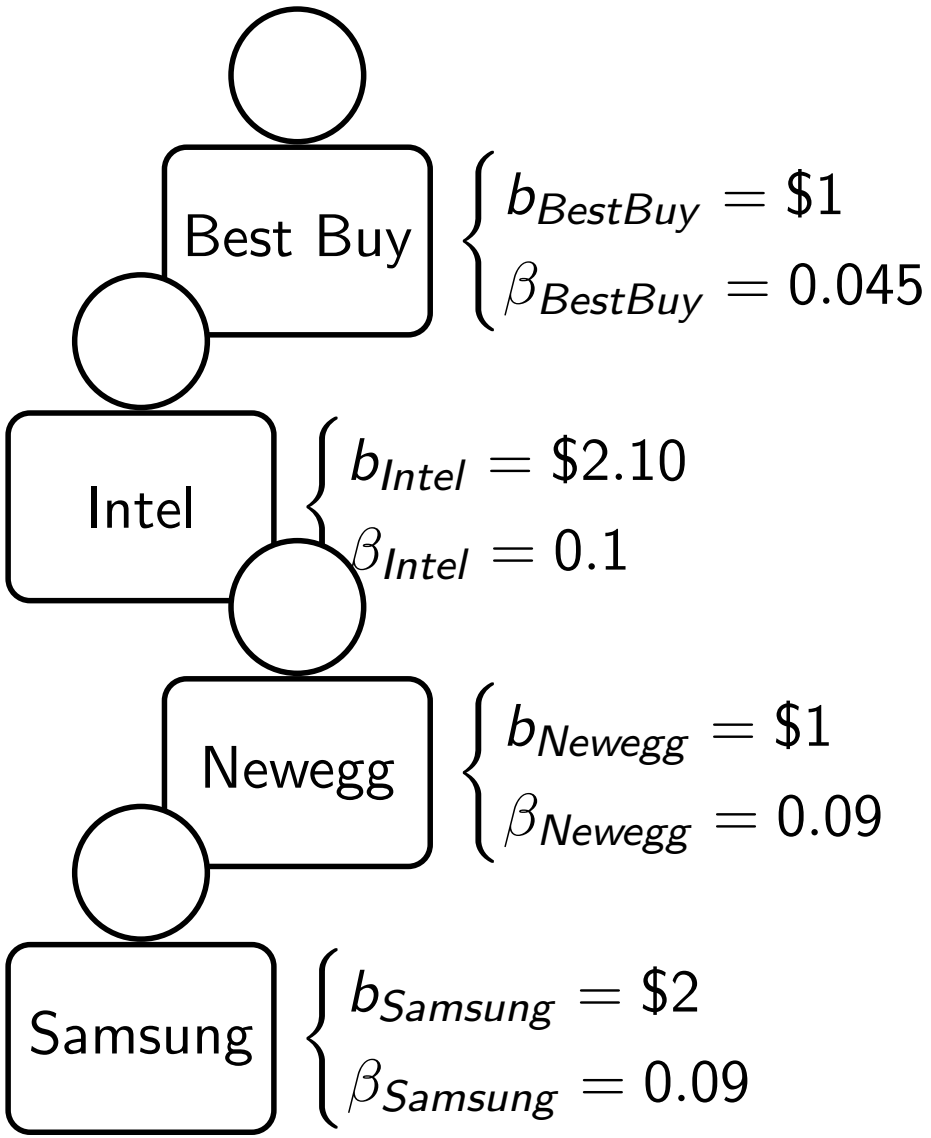


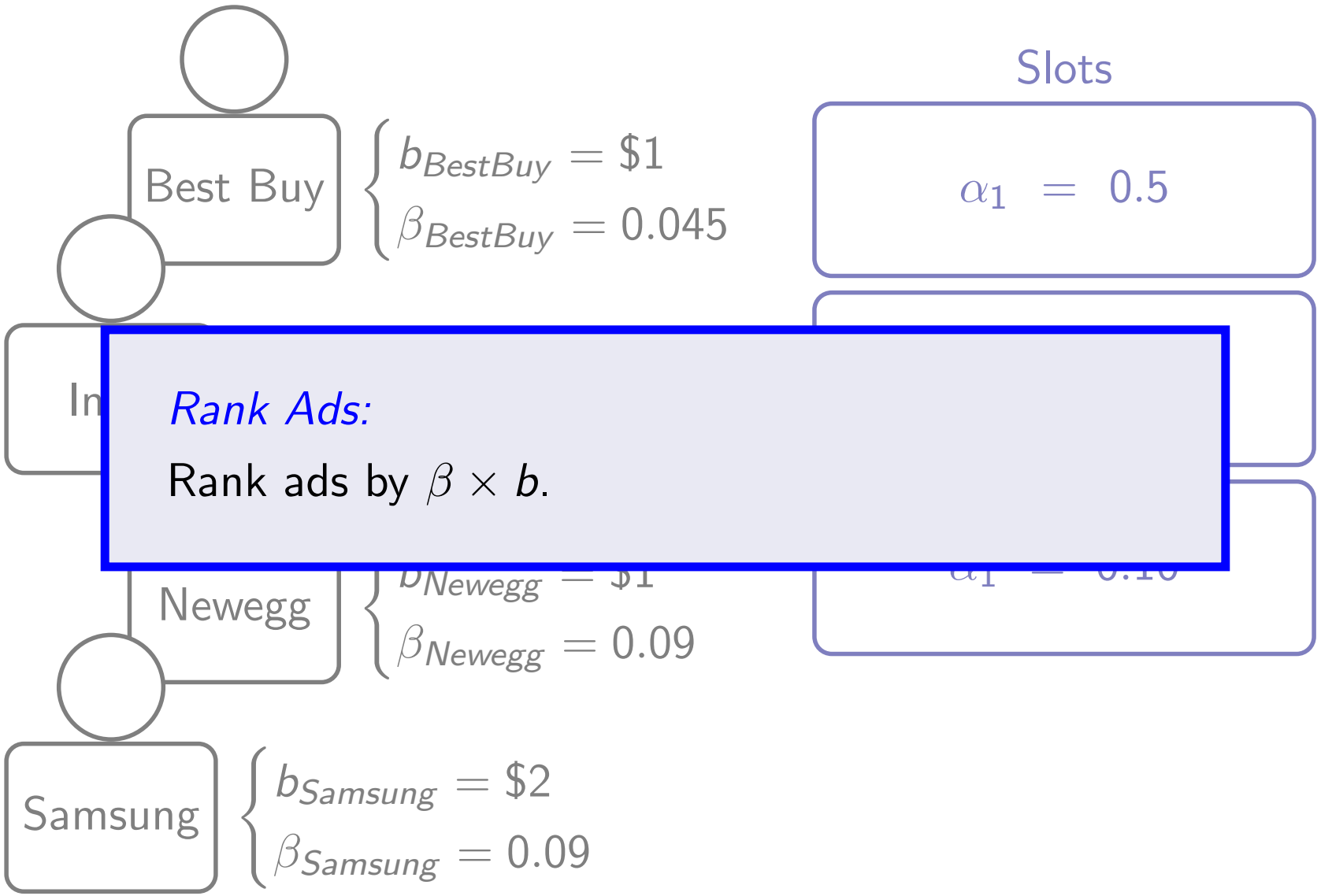
Slots

$\alpha_1 = 0.5$

$\alpha_1 = 0.25$

$\alpha_1 = 0.10$

Sponsored Search — GSP Example



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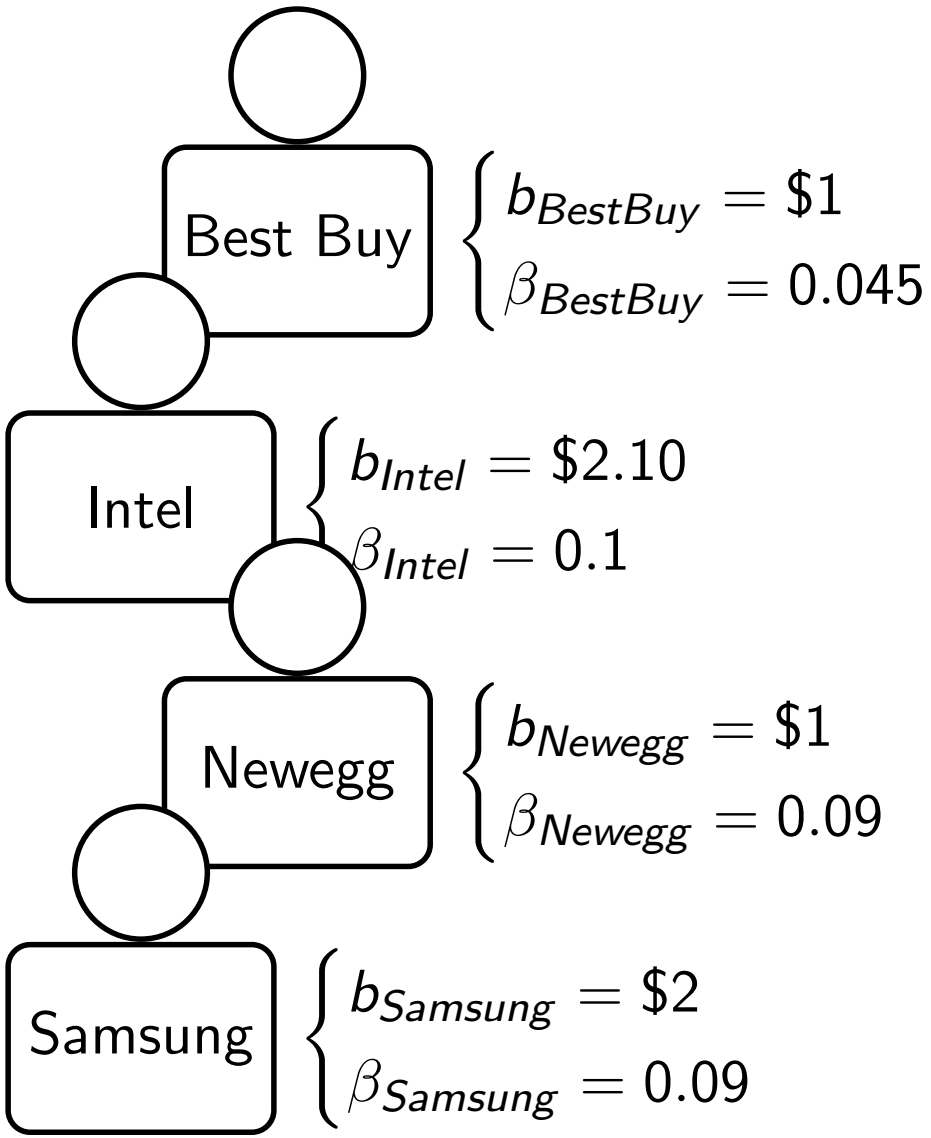

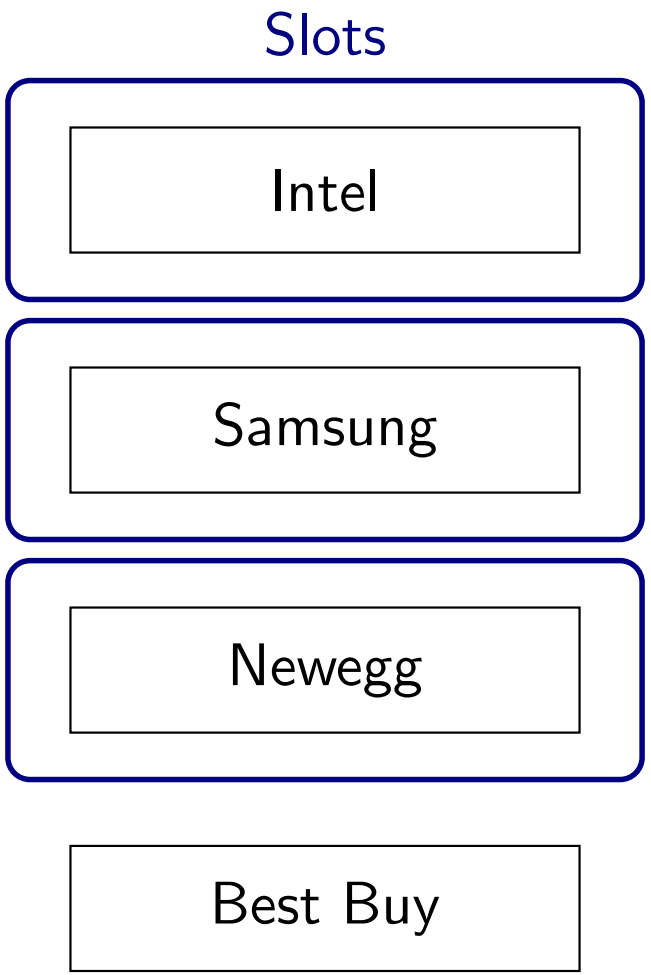

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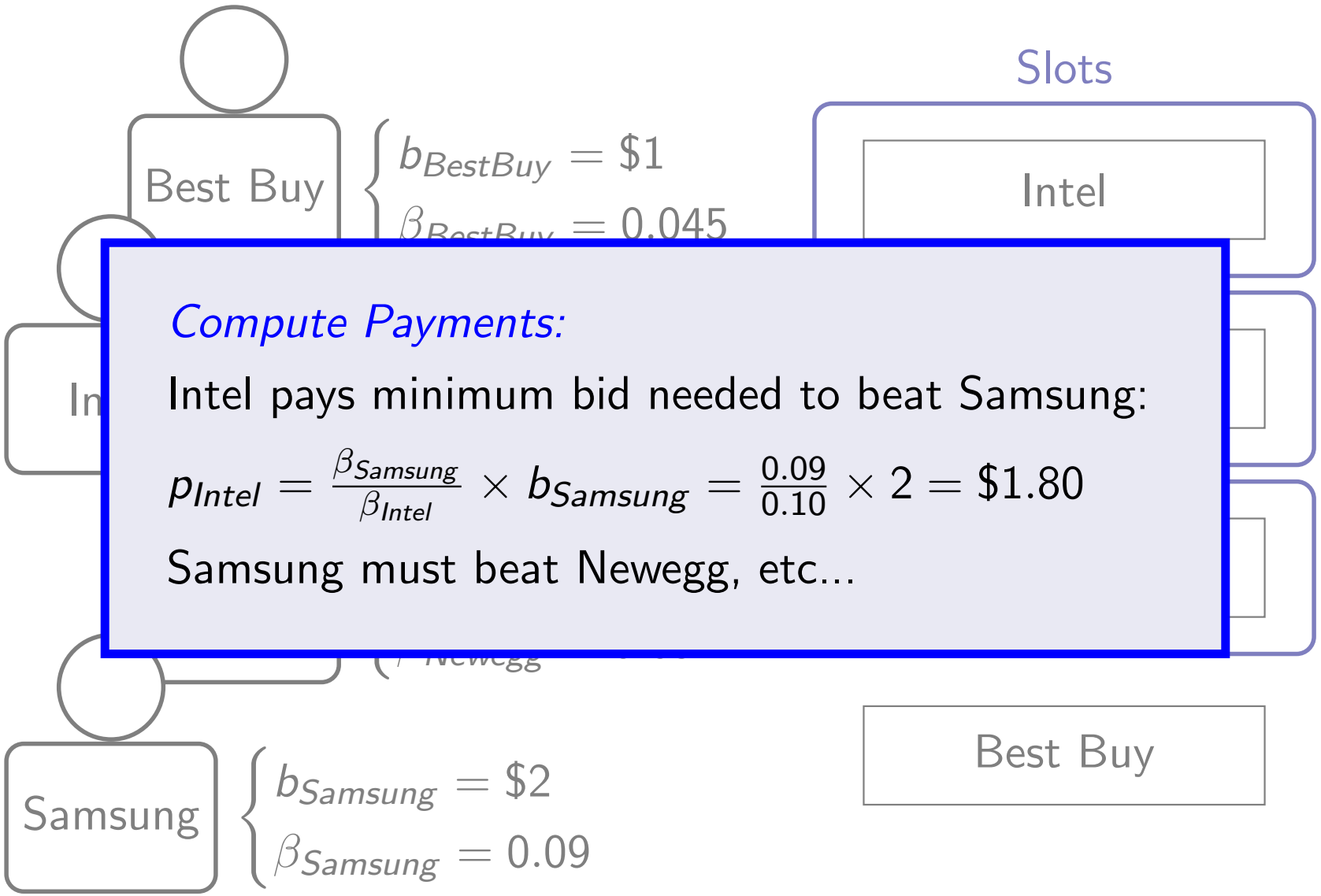

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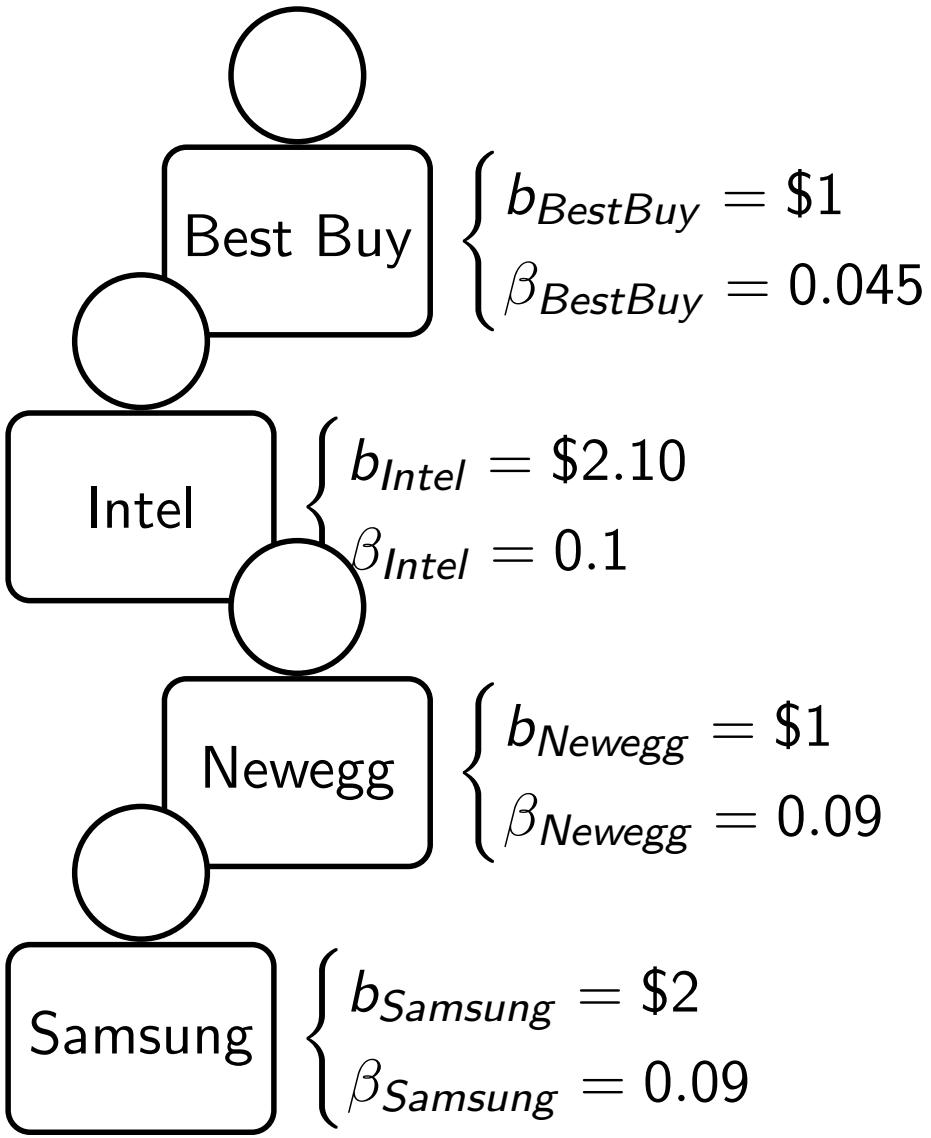


Slots

Intel
$p_{Intel} = \$1.80$

Samsung
$p_{Samsung} = \$1$

Newegg
$p_{Newegg} = \$0.50$

Best Buy

The Sponsored Search Auction: First-Price Auctions

work with Darrell Hoy and Kamal Jain

Sponsored Search — First-Price Auctions

Problem:

The GFP sponsored search auction is unstable and revenue suffers.

Sponsored Search — First-Price Auctions

Problem:

The GFP sponsored search auction is unstable and revenue suffers.

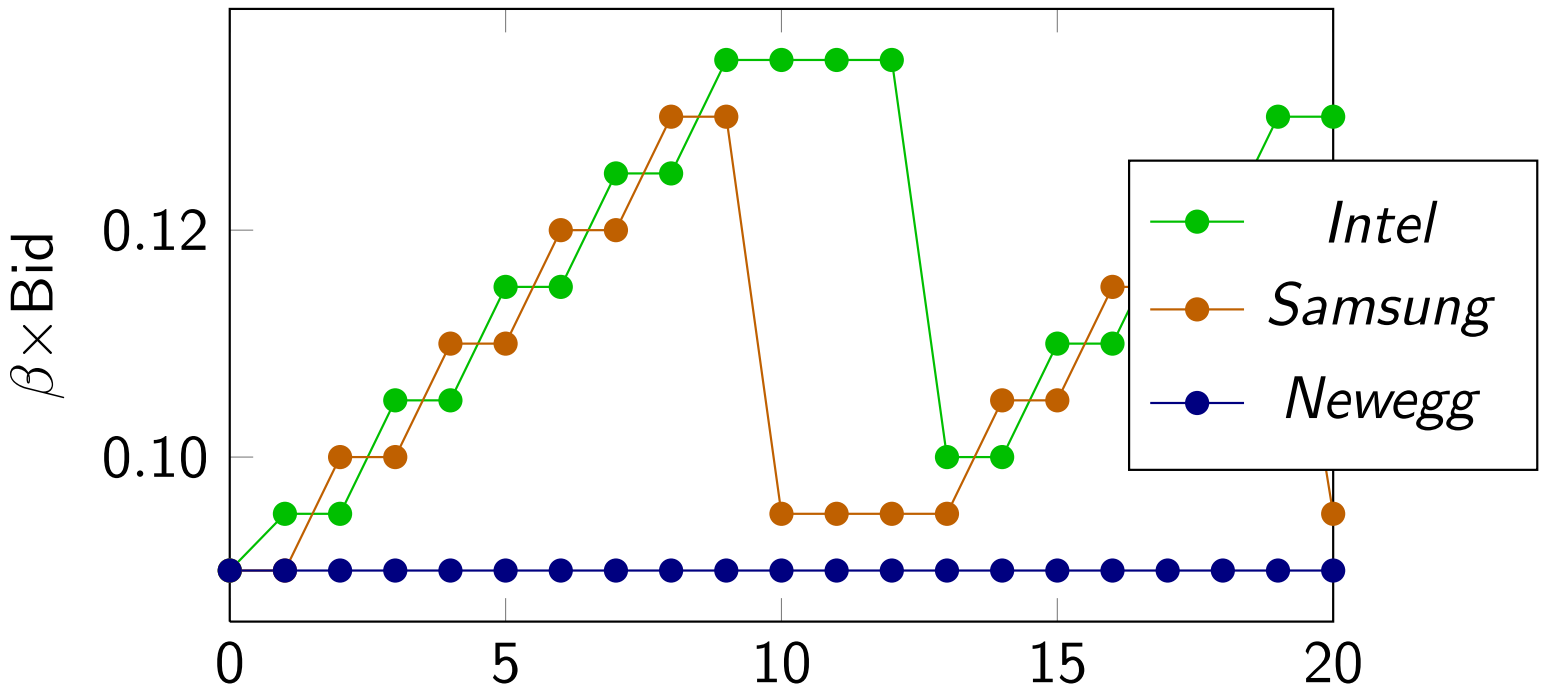
Solution (Hoy, Jain, and W):

Change the bidding language. Get:

- Strong static performance.
- Dynamic convergence.

Sponsored Search — First-Price Auctions

Instability in GFP: a Bidding War



When Intel passes $\beta_{Intel} \times b_{Intel} = \$0.18$, Samsung drops its bid...

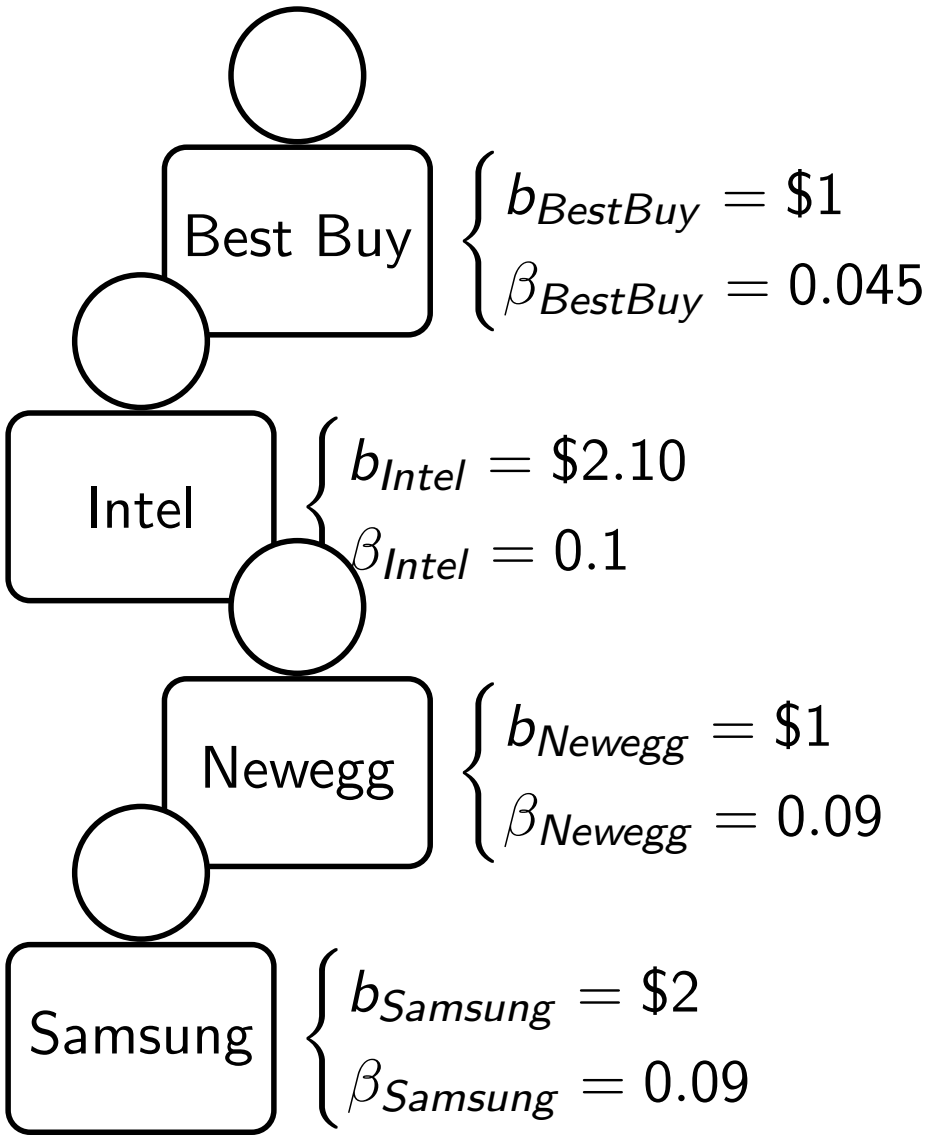
...and Intel follows.

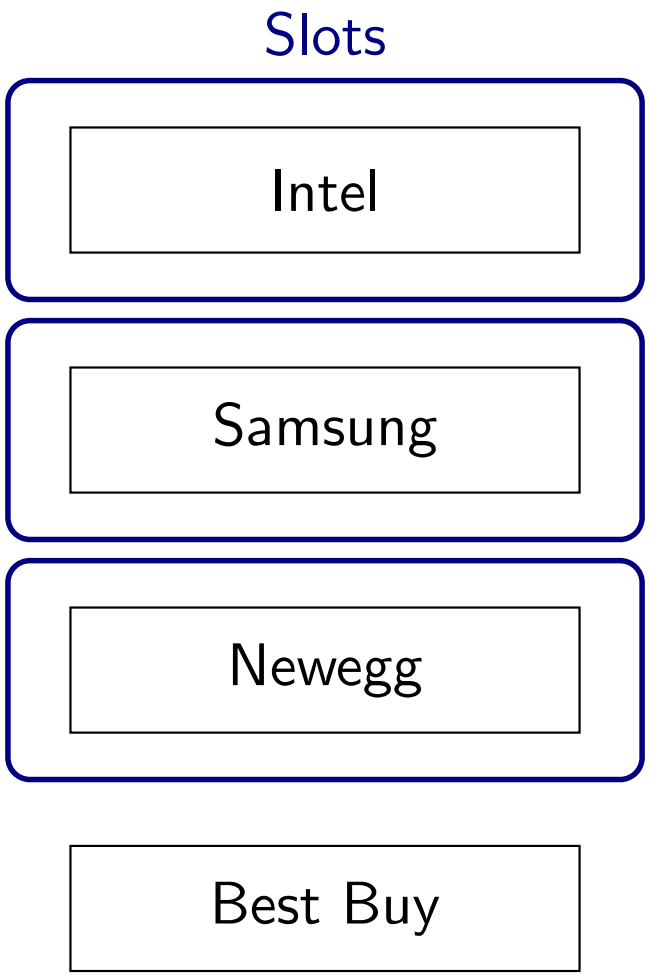
Sponsored Search — First-Price Auctions

Lahaie 2006, Edelman and Ostrovsky 2007:

GFP does not have a pure-strategy equilibrium.

Sponsored Search — GFP Example





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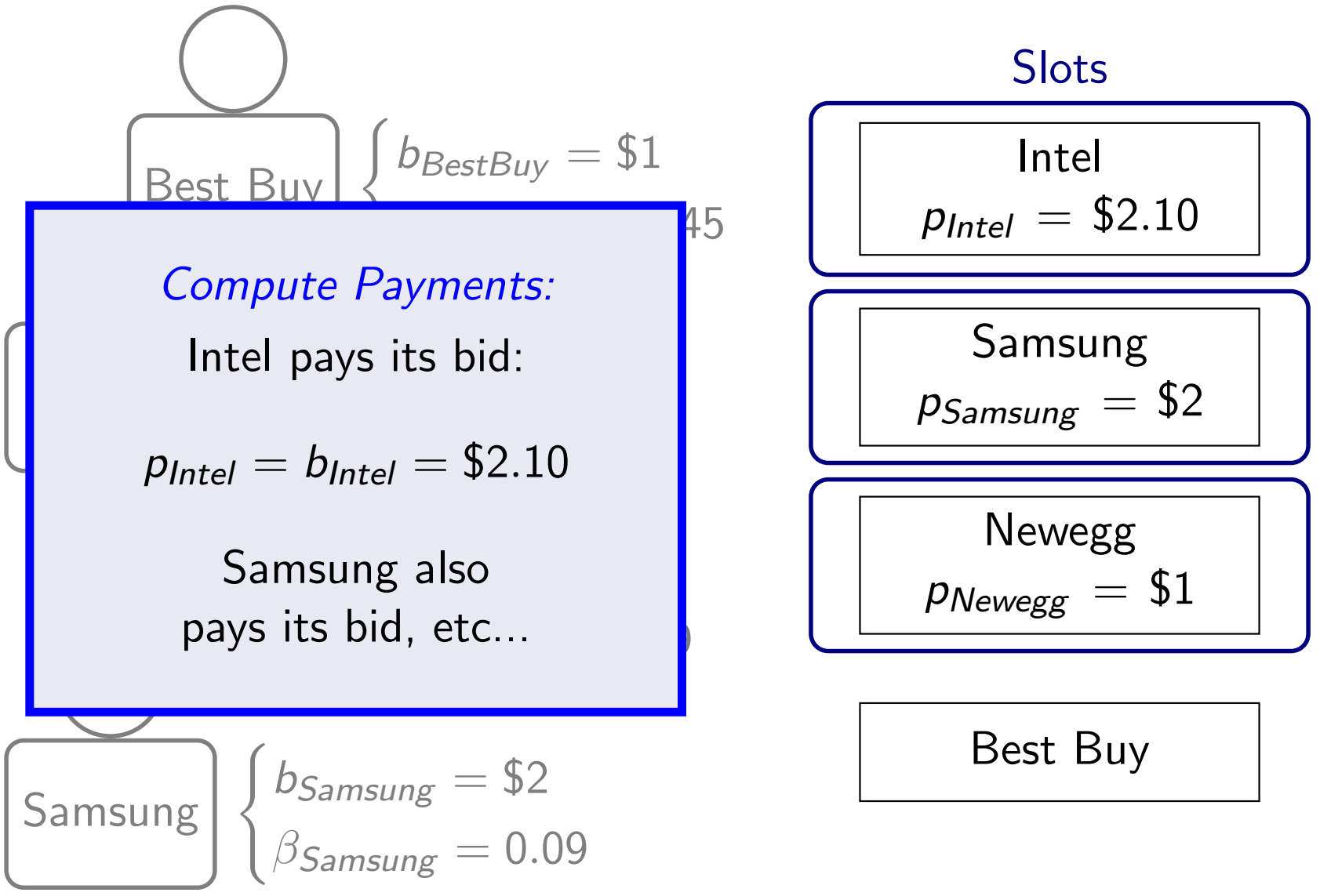


Sponsored Search — GFP Example

Equilibria cannot exist:

(a) Samsung must bid minimum to beat Newegg.

45

Samsung

$\beta_{Samsung} = 0.09$

Slots

Intel

Samsung
$p_{Samsung} = \$1$

Newegg
$p_{Newegg} = \$1$

Best Buy

Sponsored Search — GFP Example

Equilibria cannot exist:

(a) Samsung must bid minimum to beat Newegg.

(b) Intel must bid minimum to beat Samsung.

45

Samsung

$\beta_{Samsung} = 0.09$

Slots

Intel

$p_{Intel} = \$0.90$

Samsung

$p_{Samsung} = \$1$

Newegg

$p_{Newegg} = \$1$

Best Buy

Sponsored Search — GFP Example

Equilibria cannot exist:

(a) Samsung must bid minimum to beat Newegg.

(b) Intel must bid minimum to beat Samsung.

(c) Given (a) and (b), Samsung should raise its bid to beat Intel.

45

Samsung

$\beta_{Samsung} = 0.09$

Slots

Intel

$p_{Intel} = \$0.90$

Samsung

$p_{Samsung} = \$1$

Newegg

$p_{Newegg} = \$1$

Best Buy

Sponsored Search — First-Price Auctions

Lahaie 2006, Edelman and Ostrovsky 2007:

GFP does not have a pure-strategy equilibrium.

Consequence:

Some bidder always has an incentive to change her bid.

Edelman and Ostrovsky 2007:

VCG would generate more revenue than sawtooth behavior.

Sponsored Search — First-Price Auctions

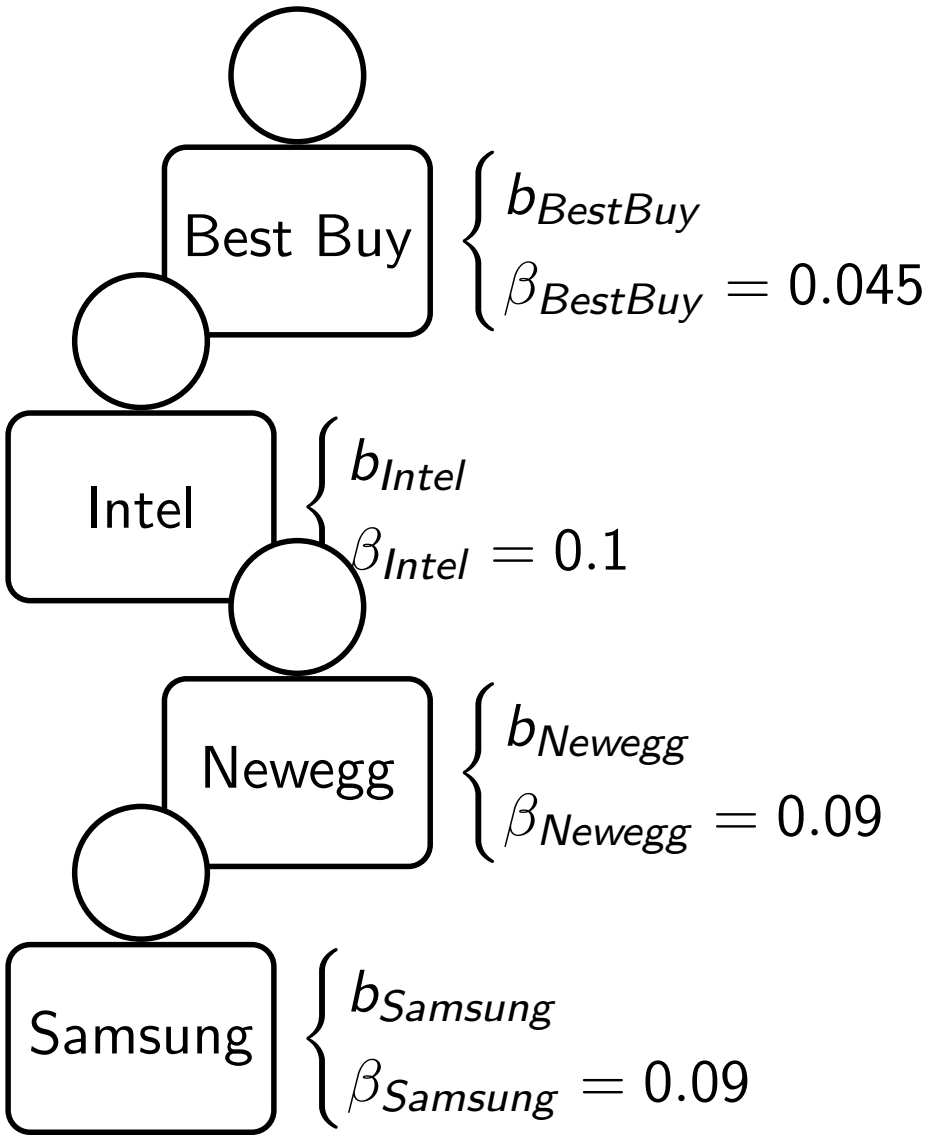
Observation:

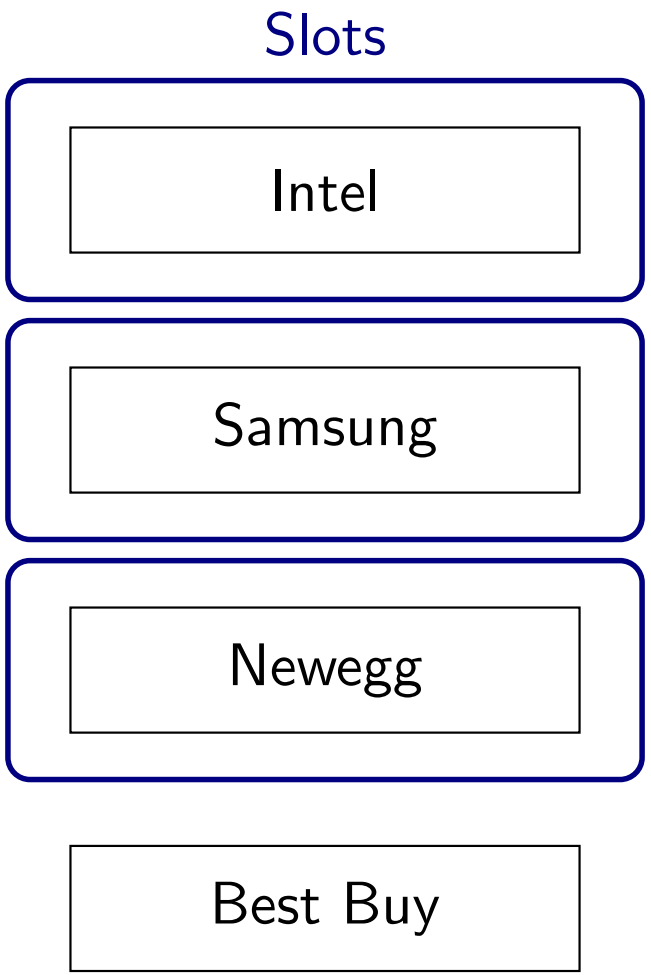
Equilibria exist if advertisers can place complex bids.

For example....

Pure-strategy equilibria exist if advertisers can place a separate bid for each slot. (e.g. Bernheim and Whinston 1986)

Sponsored Search — GFP Example





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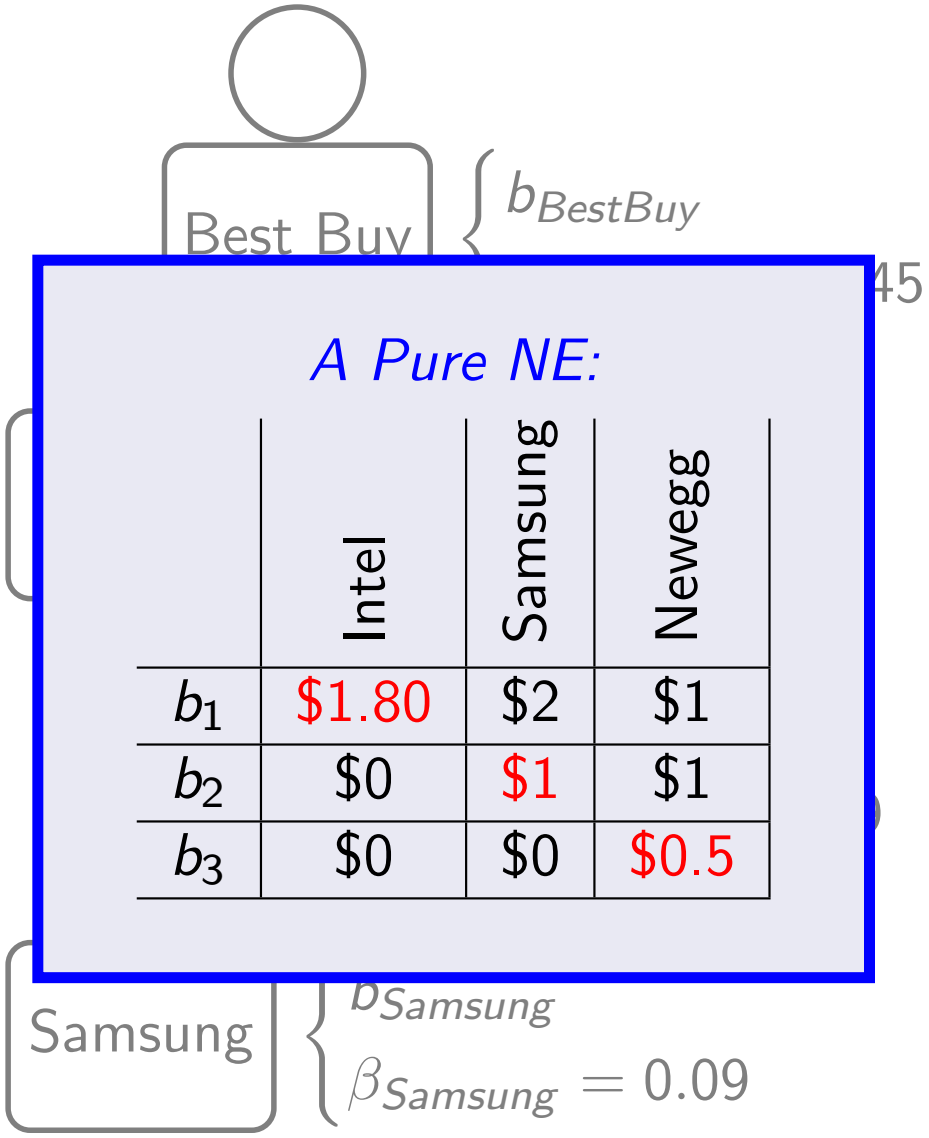
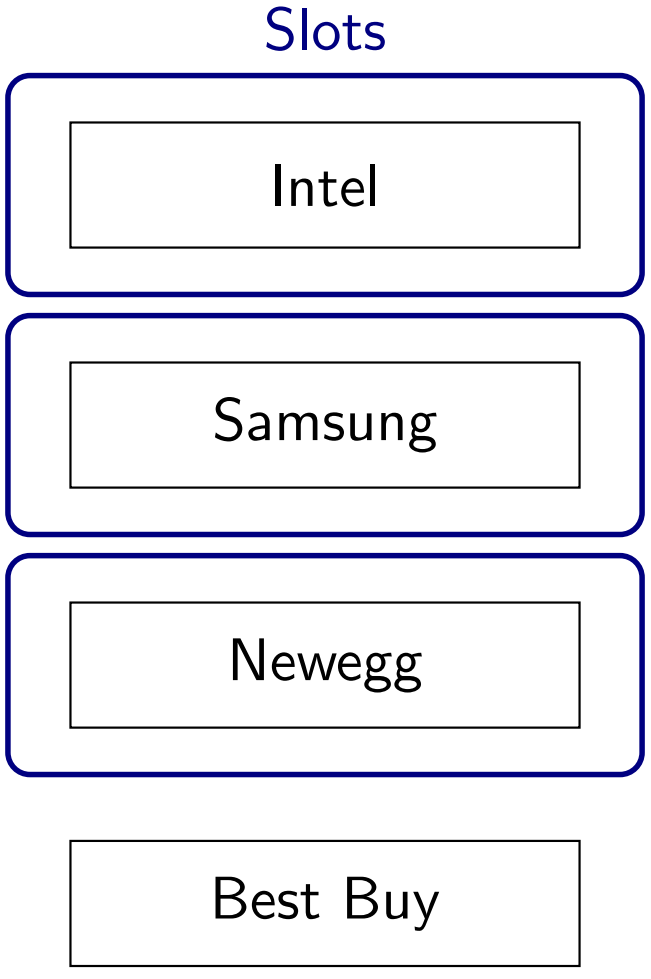


A Pure NE:

	Intel	Samsung	Newegg
b_1	\$1.80	\$2	\$1
b_2	\$0	\$1	\$1
b_3	\$0	\$0	\$0.5



Sponsored Search — First-Price Auctions

Question:

How complex must the bidding language be?

Hoy, Jain, and W:

Language only needs to encode a bidder's per-click value v_i and the final utility π_i she requests.

Sponsored Search — First-Price Auctions

Definition (Utility-Target Ad Auction)

Bids: Player i bids (x_i, π_i)

- x_i represents per-click value
- π_i represents target utility

Payments: For each slot assignment $j(i)$, define payments:

$$p_i = \max\left(x_i - \frac{\pi_i}{\alpha_{j(i)}\beta_i}, 0\right)$$

Outcome: Choose the ranking that maximizes revenue:

$$\text{argmax}_{j(\cdot)} \sum_i \alpha_{j(i)}\beta_i p_i \quad .$$

Sponsored Search — First-Price Auctions

Definition (Utility-Target Auction)

Bids: Player i bids (x_i, π_i)

- $x_i \in V_i$ represents player i's valuation function
- $\pi_i \in \Re^+$ represents target utility

Payments: For each outcome o define payments:

$$p_i(o) = \max(v_i(o) - \pi_i, 0)$$

Outcome: Choose the outcome o^* that maximizes revenue:

$$o^* = \text{argmax}_o \sum_i p_i(o) \ .$$

Note: $u_i(o) = v_i(o) - p_i(o) = \min(\pi_i, v_i(o))$

Sponsored Search — First-Price Auctions

Question:

Why do we need π_i? The bid (x_i, π_i) specifies a payment function $p_i(o)$, so why is it not enough to consider bids of the form $(p_i, 0)$?

Answer:

It may not be possible to bid $(p_i, 0)$, e.g. GFP requires that x_i represent the same value for a click on each slot, whereas p_i can encode different payments (values) for each slot.

Sponsored Search — First-Price Auctions

Definition (Quasi-truthfulness)

A utility-target auction is *quasi-truthful* if i never has an incentive to misreport her valuation, i.e. to report $x_i \neq v_i$.

Theorem (Hoy, Jain, and W)

For any bid (x_i, π_i) that generates utility u_i, the bid (v_i, u_i) also generates utility u_i.
As a consequence, the utility-target auction is quasi-truthful.

Sponsored Search — First-Price Auctions

Definition (Cooperative Envy-Freeness (CEF))

A set of payments $p_i(o)$ and an outcome o^* are *cooperatively envy-free (CEF)* if no coalition is collectively willing to increase bids so an alternate outcome o wins, i.e.

$$\sum_i \max((v_i(o) - p_i(o)) - (v_i(o^*) - p_i(o^*)), 0) \leq \sum_i p_i(o^*) - p_i(o)$$

for all outcomes o.

Remark: This weaker than equilibrium concepts like strong Nash equilibrium, group strategyproofness, the core, etc.

Sponsored Search — First-Price Auctions

Theorem (Hoy, Jain, and W)

Every utility-target auction has a quasi-truthful CEF pure-strategy equilibrium. Any such equilibrium...

- *is efficient (maximizes bidder welfare), and*
- *has revenue at least as large as the VCG mechanism.*

Remark: This is analogous to profit-target bidding in package auctions (Bernheim and Whinston 1986, Milgrom 2004,...).

Sponsored Search — First-Price Auctions

Issue:

The utility-target auction generalizes...

...but beating VCG may not mean much.

VCG's Downfall:

I have a plot of land to sell two ways:

- Alice is willing to pay $20k.
- Bob, Charlie, and Dave are willing to pay $33k total ($11k each) and share the property.

Since B/C/D wins even if one person drops out, VCG charges $0.

Sponsored Search — First-Price Auctions

The Second-Price Threat Benchmark:

$$\gamma = \max_{o} \sum_{i} \max(v_i(o) - v_i(o^*), 0)$$

Idea: How much would bidders be willing to pay (in total) to ensure that o was chosen instead of o^*? (e.g. \$20k)

Theorem (Hoy, Jain, and W)

Revenue of the utility-target auction in any envy-free equilibrium is at least γ.

Sponsored Search — First-Price Auctions

Is equilibrium a credible prediction?

Yes, by a dynamic argument!

Dynamic Properties of Bidder Behavior:

1. A "loser" will only decrease the utility she requests.
   A "winner" will only increase the utility she requests.
2. A loser will always try to raise her bid.
3. Losers are less patient than winners.
4. A loser's patience is inversely related to her requested utility.

Theorem (Hoy, Jain, and W)

- *(1)-(2) $\Rightarrow$ bids eventually exceed revenue benchmarks.*
- *(1)-(4) $\Rightarrow$ bids converge to the egalitarian equilibrium.*

Sponsored Search — First-Price Auctions

Lessons:

- The complexity of bids is important:
  - Bids may be more complex than valuations...
  - ...but need not be too much more complex.
- Simple properties of bidder behavior have implications for convergence and revenue.

Open Questions:

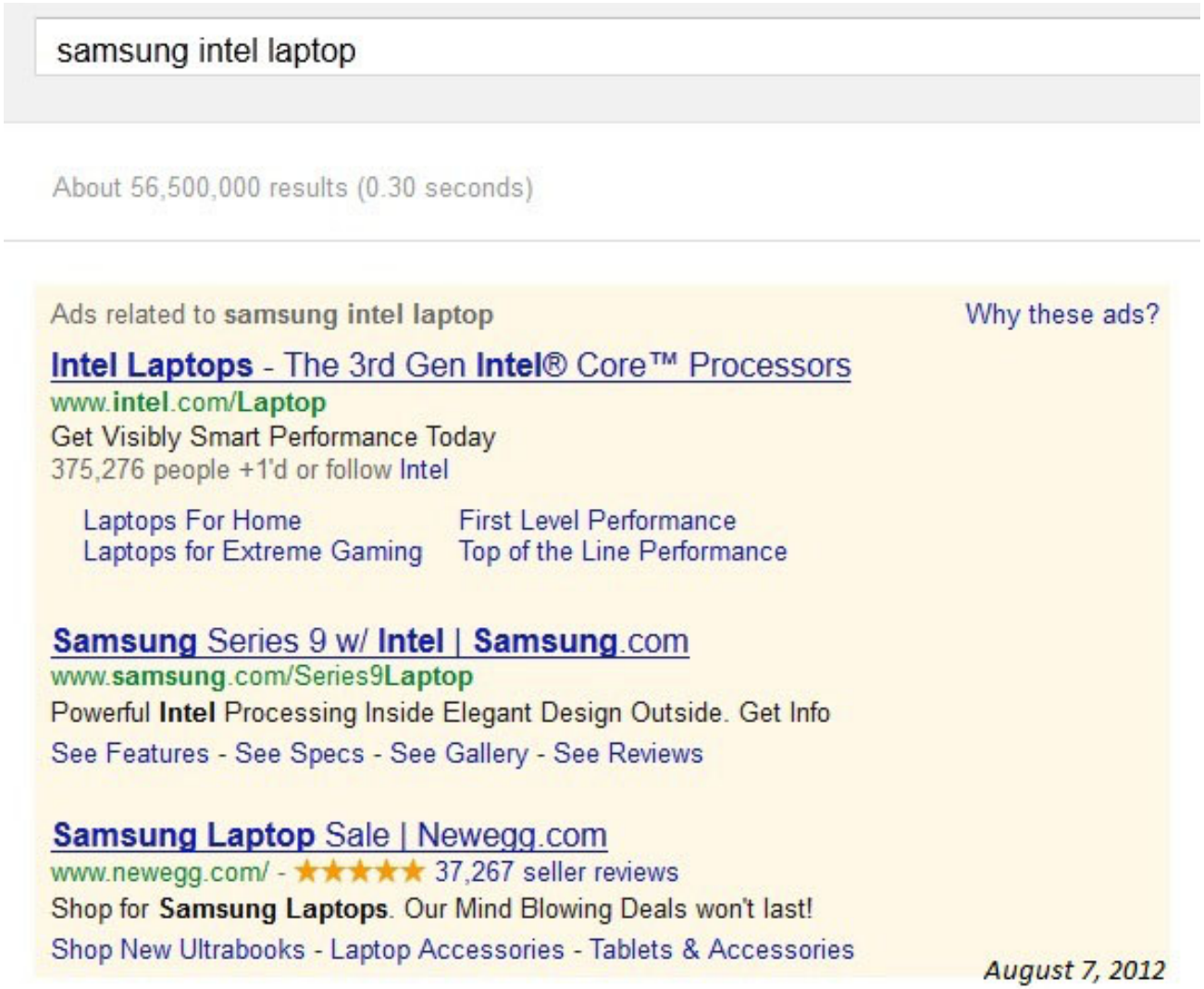
- What happens when properties (1)-(4) are relaxed?
- Does bidder behavior satisfy properties (1)-(4)?

The Sponsored Search Auction: Coopetitive Ad Auctions

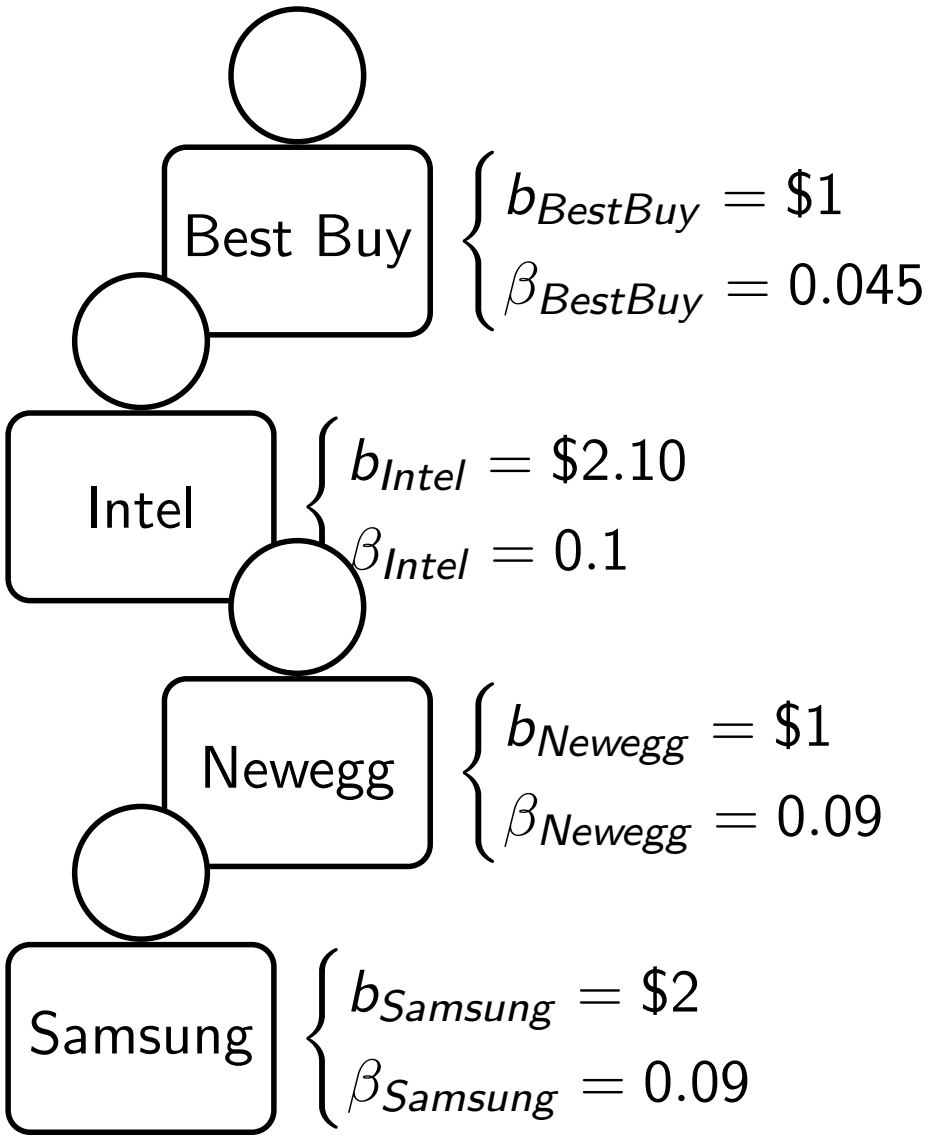
work with Darrell Hoy and Kamal Jain

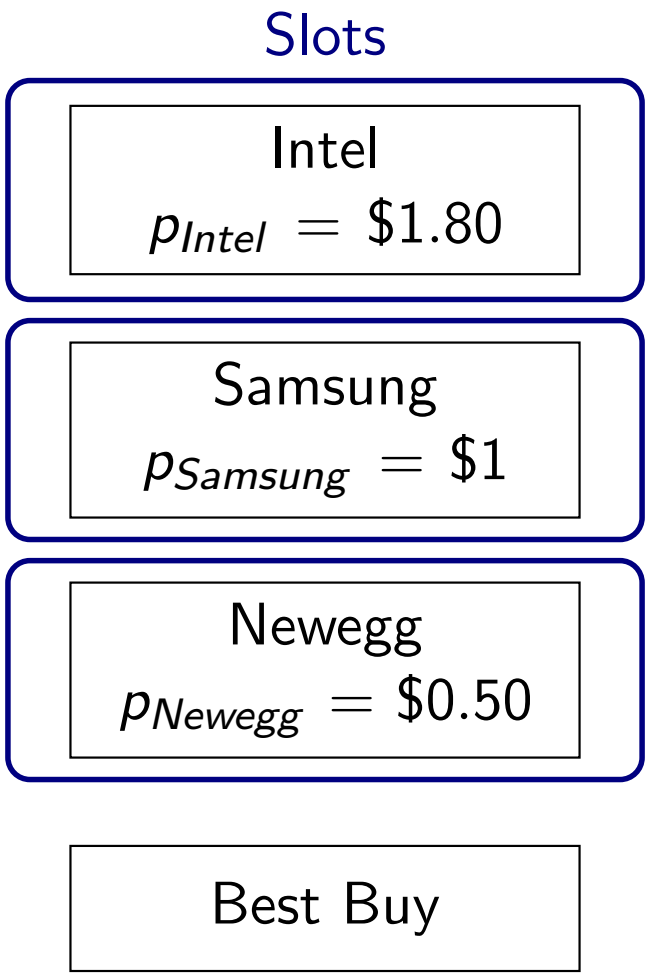
Sponsored Search — Coopetitive Ad Auctions

Question: What's wrong with this picture?



Sponsored Search — GSP Example





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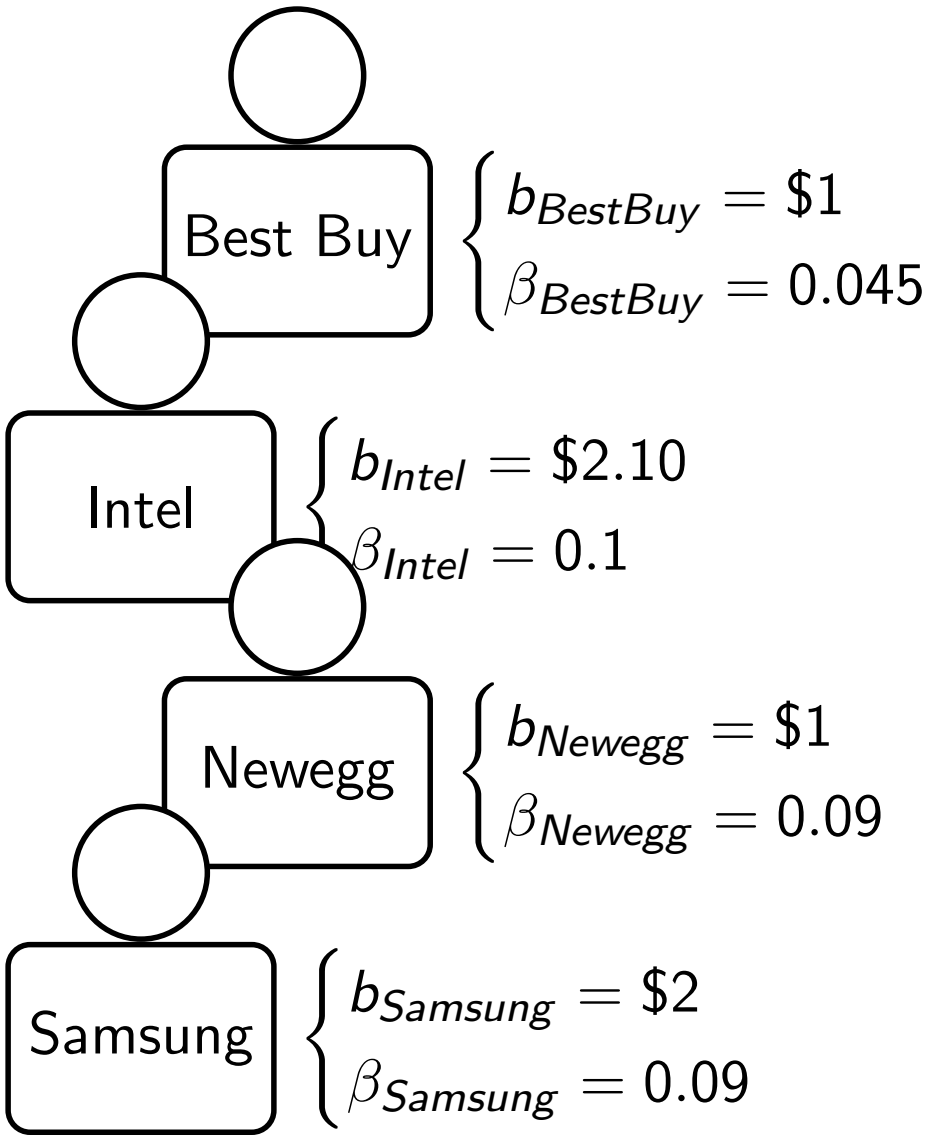
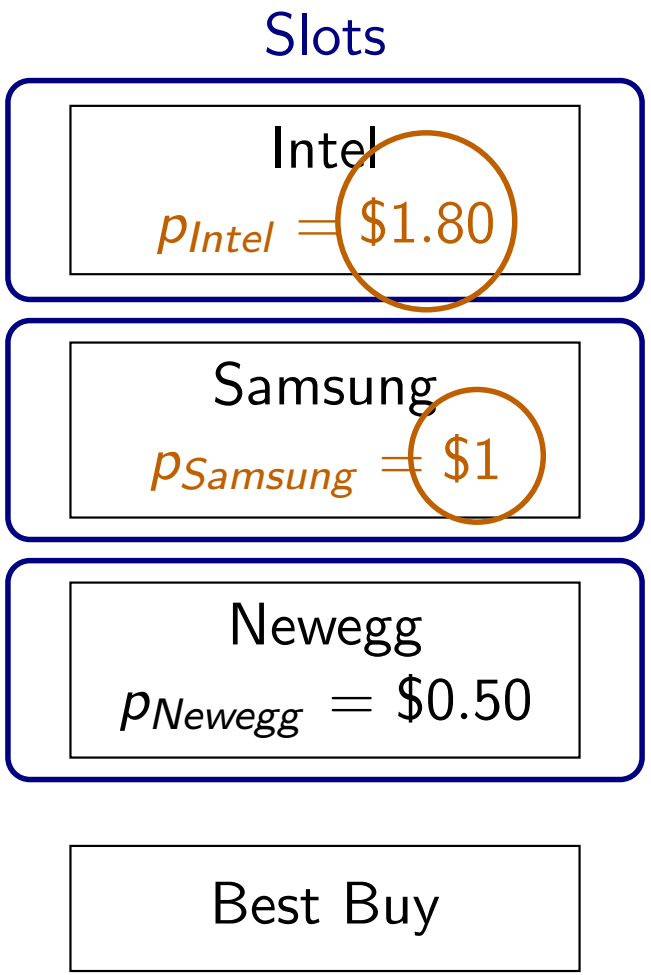
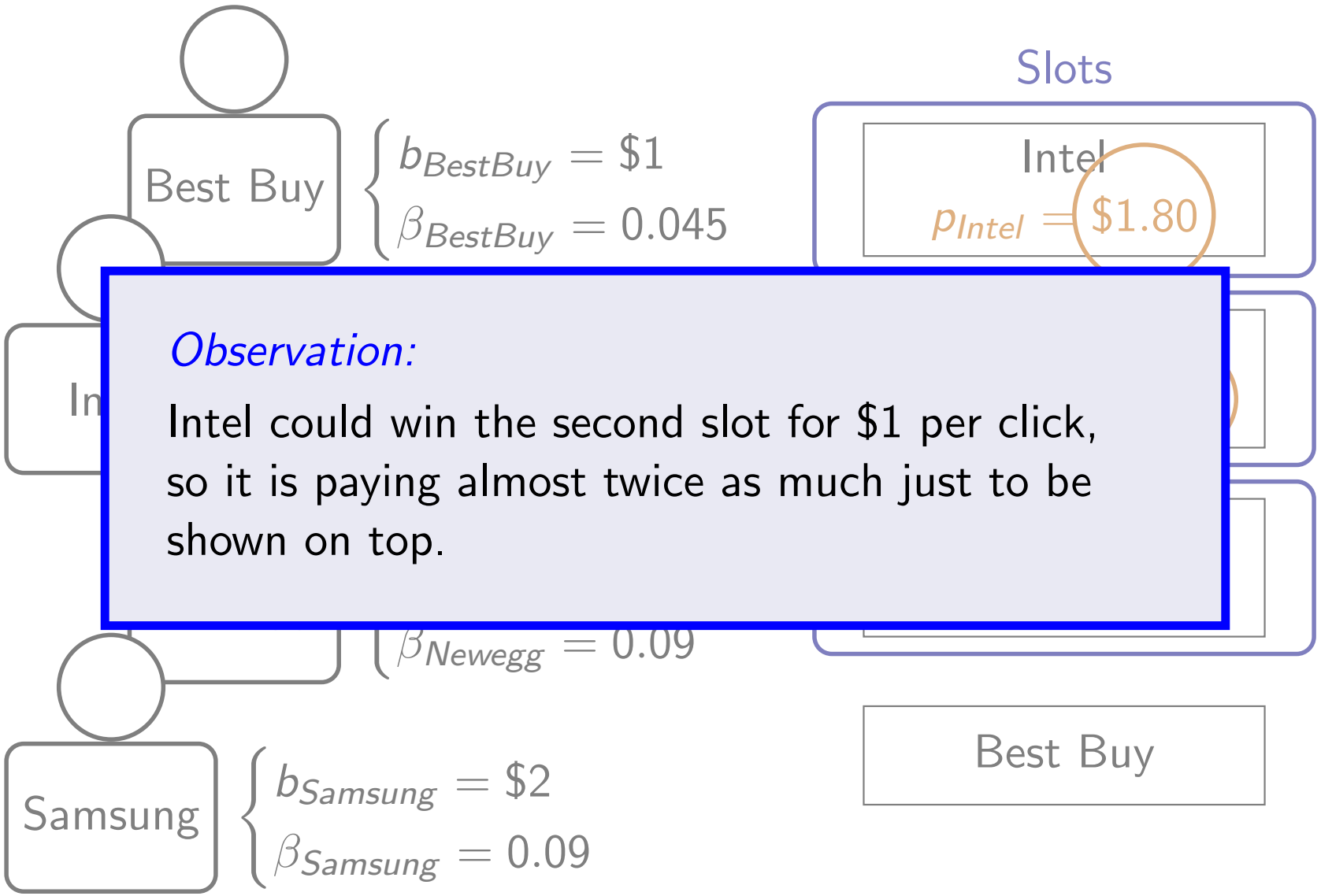




Sponsored Search — GSP Example

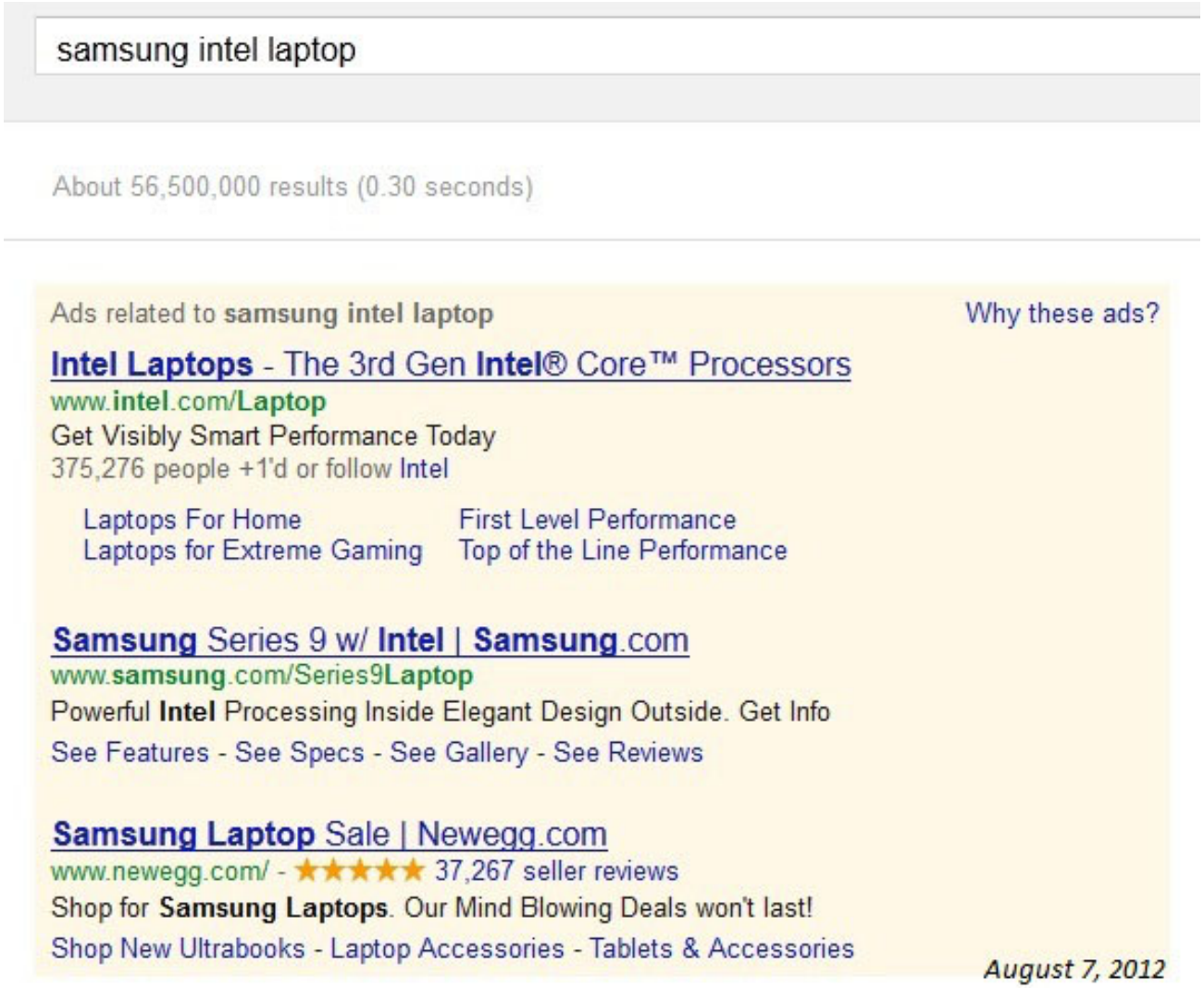


Observation:

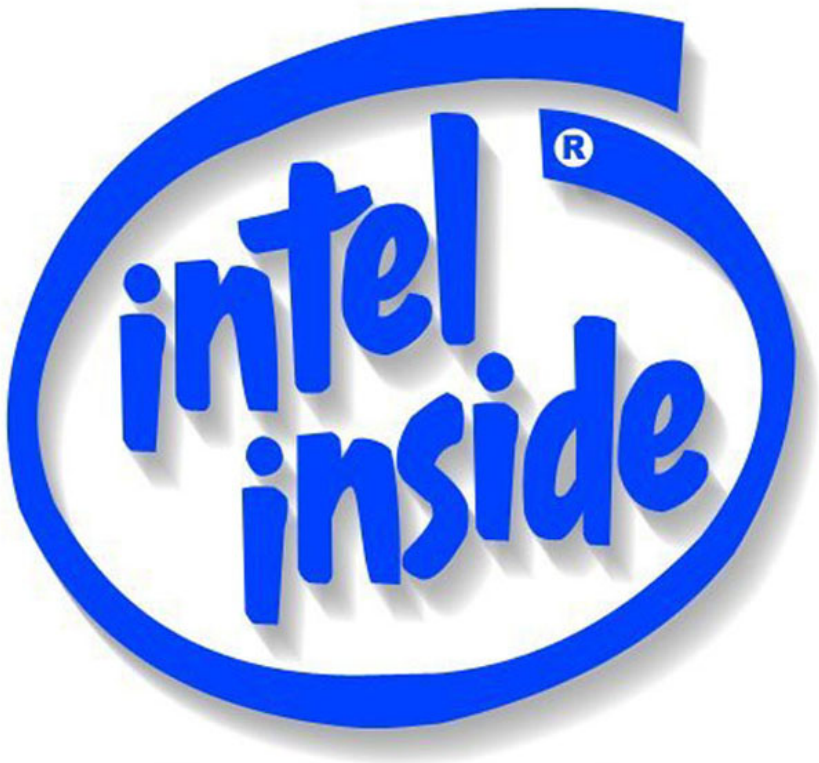
Intel could win the second slot for \$1 per click, so it is paying almost twice as much just to be shown on top.

Sponsored Search — Coopetitive Ad Auctions

Answer: Intel paid too much!



Sponsored Search — Coopetitive Ad Auctions



Sponsored Search — Coopetitive Ad Auctions

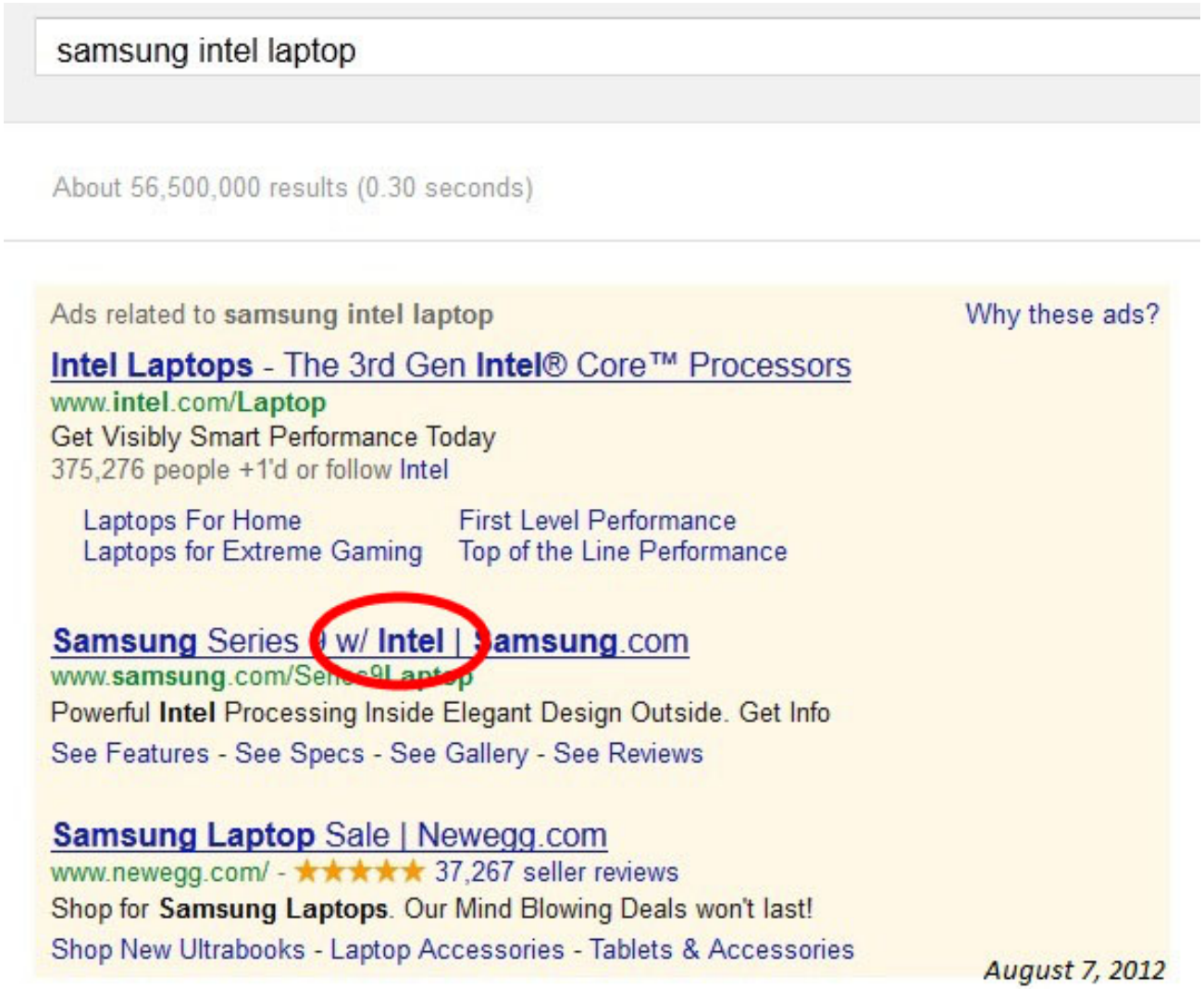
Intel Inside:

Intel pays fraction of advertising costs...

...as long as the advertiser includes "Intel Inside" branding.

Sponsored Search — Coopetitive Ad Auctions

Is this part of "Intel Inside"?



Sponsored Search — Coopetitive Ad Auctions

If this is Intel Inside...

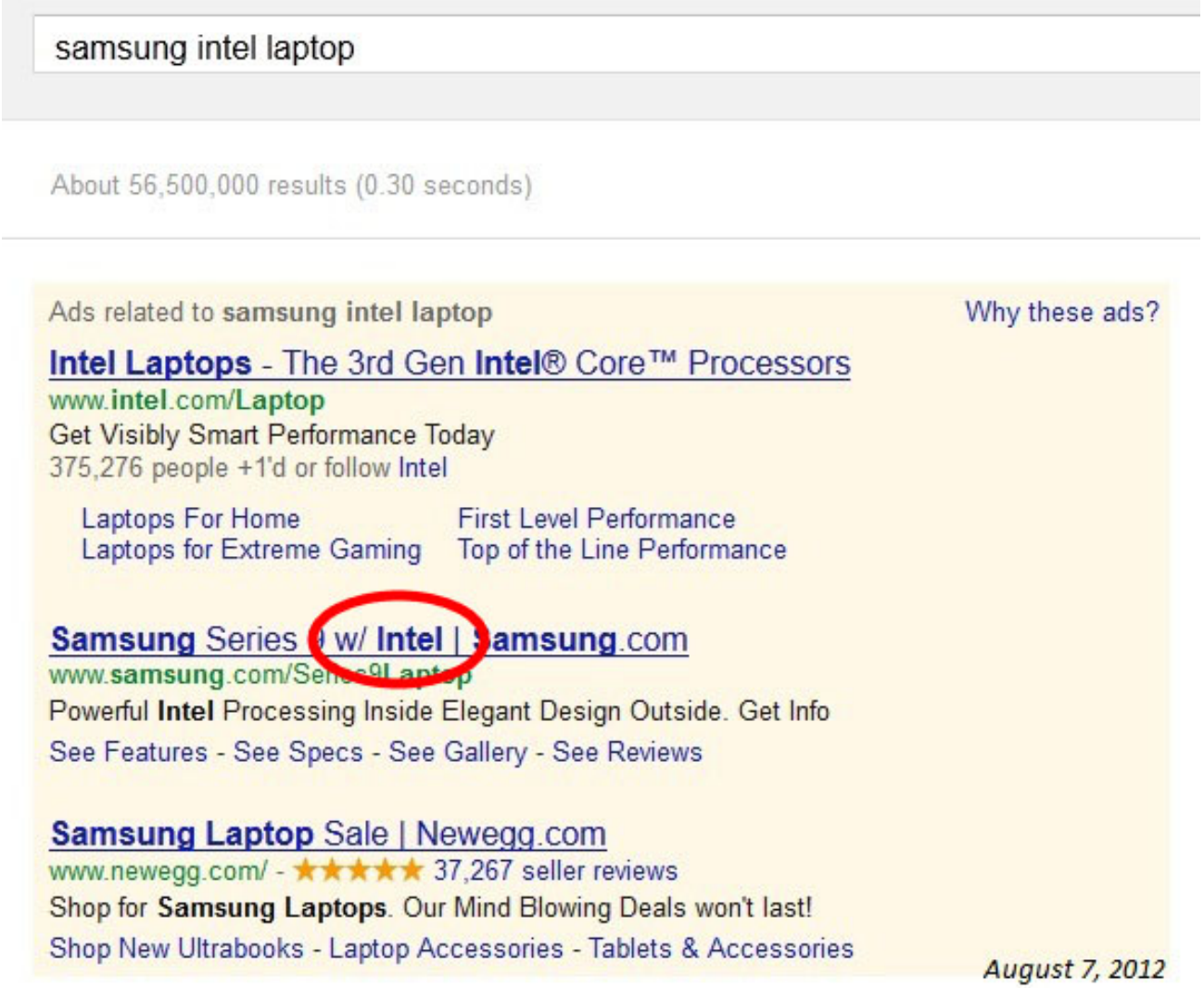
Intel will pay some of Samsung's costs...

...so Samsung bids higher...

...so Intel's payment goes up!

Sponsored Search — Coopetitive Ad Auctions

Answer 2: Intel may be competing with itself!



Sponsored Search — Coopetitive Ad Auctions

Cooperative Advertising:

Estimated $15B spent in 2000 in US alone. (Nalger 2006)

Prior work...

Modeled as Stackelberg game, upstream manufacturer (e.g. Intel) is first-mover. (survey by He et al. 2007)

New question:

How should an advertising platform sell ads when a single ad can benefit many advertisers?

Sponsored Search — Coopetitive Ad Auctions

Goal:

Auction format that maintains competition between ads while encouraging cooperation within a single ad.

...i.e. we want coopetition.

Sponsored Search — Coopetitive Ad Auctions

Possible Solutions:

Keep the status quo...

...but we show cooperation may fall apart, hurting performance.

Sponsored Search — Coopetitive Ad Auctions

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Open Question: Better solutions?

Complexity Equilibria

work with Christos Papadimitriou

Complexity Equilibria

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Complexity Equilibria

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Are there "pseudo-equilibria" that are stable because it is too hard to find a good deviation?

Complexity Equilibria

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Papadimitriou and W (2011):

...in markets with economies of scale in production, yes!

Complexity Equilibria

Definition (Market Equilibrium, paraphrased)

A *market equilibrium* is a set of prices where supply equals demand when people selfishly optimize their own behavior.

Complexity Equilibria

Definition (Market Equilibrium, paraphrased)

A *market equilibrium* is a set of prices where supply equals demand when people selfishly optimize their own behavior.

Theorem (First Welfare Theorem)

In market equilibrium, it is impossible to make one person happier without hurting someone else.

- This is called Pareto efficiency.
- Pareto efficiency is the gold standard for acceptable economic outcomes.

Complexity Equilibria

Issue:

Arrow and Debreu proved that equilibria always exist assuming *no* economies of scale.

...with economies of scale, equilibria may not exist.

Guessnerie (1975):

Can we at least achieve Pareto Efficiency through a decentralized process?

Complexity Equilibria

Observation:

Pareto efficiency is a notion of stability where a "good deviation" means improving for someone without hurting anyone.

...i.e. a Pareto improvement.

Theorem (Papadimitriou and W, 2011)

In a family of markets with economies of scale in production, there exist complexity equilibria from which it is NP-hard to compute a Pareto improvement.

Proof Intuition:

A factory may produce cars or TV's, but cannot produce both well, i.e. producers may face discrete choices.

Complexity Equilibria

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Complexity Equilibria

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NP-hardness implies it is intractable to determine whether a Pareto-improvement exists...

...so what?

Complexity Equilibria

Economies of scale are huge...

Google, Facebook, Microsoft, etc. have huge economies of scale, and the internet is just making them bigger.

Startups...

Startups try to solve

$$\textsc{NextBigThing}(x) = \textsc{True} \ .$$

Startups spring up all the time, surviving if they achieve sufficient scale.

Complexity Equilibria

Lessons:

- Compexity theory (NP-hardness) gives us a generalization of stability.
- Economies of scale are real, and may lead to complexity equilibria.

Open Question:

Can we demonstrate complexity equilibria in real settings, like startups or power markets?

Conclusion

Recap — Computation and Economics

Sponsored Search Auctions

- *First-Price Auctions:*
  A more complex bidding language improves stability, and dynamic arguments offer alternative performance guarantees.
- *Coopetitive Ad Auctions:*
  Recognizing complexity may be important for performance.

Market Equilibria

- *Complexity Equilibria in Markets*
  Computational complexity begets stability.

Thank You.